In order to sit the examination you must be enrolled in the course. No aids allowed. Use the papers provided by the department and write on one side of each sheet only. Fill in the cover completely and write your initials on each sheet. Write legibly. Give concise and short arguments and draw figures when applicable.

1. The vectors \( u_1 = (2, 3, 4), u_2 = (1, 2, 3) \) and \( u_3 = (3, 2, 2) \) are given with respect to a basis \( e_1, e_2, e_3 \) in the 3-dimensional space. Show that \( u_1, u_2 \) and \( u_3 \) form a basis in the 3-dimensional space and determine the coordinates of \( e_1, e_2 \) och \( e_3 \) with respect to this new basis.

2. Let \( \ell_1 \) be the intersection line between the planes \( x + y - z = 2 \) and \( x - y + z = -4 \). Let \( M \) be the plane through the point \((0, 3, -2)\) that is orthogonal to the line \( \ell_2 \) \( (x, y, z) = (2t, 1 + 3t, 1 - 3t), t \in \mathbb{R}. \) (ON-system assumed.)
   a) Show that \( \ell_1 \) is parallel to \( M \) and determine the distance between \( \ell_1 \) and \( M \).
   b) Do the lines \( \ell_1 \) and \( \ell_2 \) intersect? Motivate your answer!

3. Let \( S \) be the sphere of radius 5 centered at the point \((2, 3, -1)\). Let \( M_1 \) and \( M_2 \) be two parallel planes with normal vector \((1, 2, -2)\), that lie at distance 3 to the centre of the sphere. Let now \( C_1 \) and \( C_2 \) be the circles of intersection between the sphere and the planes \( M_1 \) and \( M_2 \) respectively. Determine the equations of the two planes and the midpoints of the circles \( C_1 \) and \( C_2 \). (ON-system assumed.)

4. Let \( \pi \) be the plane through the point \((2, 1, 1)\) orthogonal to both planes \( 2x+y+z+1 = 0 \) and \( x - y + z = 1 \). Determine a positively oriented orthonormal basis \( u_1, u_2, u_3 \) for the 3-dimensional space such that \( u_1 \) and \( u_2 \) are parallel to the plane \( \pi \). Determine the orthogonal projection of the vector \( u = (1, 1, 1) \) on \( u_3 \). (Positively oriented ON-system assumed.)

5. Let \( a \in \mathbb{R} \) and consider the matrix

\[
A = \begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 \\
0 & a^3 & a^2 & a \\
1 & a & 2 & 1
\end{bmatrix}.
\]

   a) Compute the determinant \( D(A) \) and determine those values of \( a \) such that \( A \) is invertible.
   b) Compute the inverse \( A^{-1} \) for \( a = -1 \) and solve the matrix equation \( AX + A = E \). (\( E \) denotes the unit matrix.)

Please, turn over!
6. Let $P = (3, -1, 2)$, $Q = (-1, 1, 2)$ and $R = (3, 2, -1)$ be the midpoints of the edges $AB$, $BC$ and $AC$ in a triangle $\triangle ABC$. (Positively oriented ON-system).

a) Show using vector arithmetic that $APQR$ is a parallelogram.

b) Determine a parametric equation of the line $\ell_1$ that contains the edge $AB$.

c) Let $N$ be the midpoint of the segment $PQ$ and consider the line $\ell_2$ through $N$ that is orthogonal to the plane of the triangle $\triangle ABC$. Let now $S$ be a point on $\ell_2$ that lies at distance 3 to the point $N$. Compute the volume of the tetrahedron $BPQS$. What is the relationship between the volume of the tetrahedrons $BPQS$ and $BACS$?