1. Solve the heat conduction problem
\[ \partial_t u(x, t) = \partial_x^2 u(x, t), \quad 0 \leq x \leq \pi, \quad t > 0, \]
\[ u(0, t) = u(\pi, t) = 0, \quad t > 0, \]
\[ u(x, 0) = x(\pi - x), \quad 0 \leq x \leq \pi. \]

2. Let \( u \) be the \( 2\pi \)-periodic function for which \( u(x) = x^3 - \pi^2 x \) when \(-\pi \leq x \leq \pi\).
   a) Find the Fourier series of \( u \).
   b) Compute the sum of the series
   \[ \sum_{n=1}^{\infty} \frac{1}{n^6}. \]

3. Compute the sum of the series
\[ \sum_{n=1}^{\infty} \frac{n^2 + 2n}{3^n} \]
e.g. by considering a certain power series.

4. Find a power series solution of the problem
\[ 2xy'' + (1 - 2x)y' + y = 0, \quad y(0) = 1. \]

5. Consider the sequence \( (f_n)_{n=1}^{\infty} \) where
\[ f_n(x) = \frac{x^n}{1 + x^{2n}}, \quad x \geq 0. \]
In which of the following intervals is the sequence uniformly convergent?
   a) \( \left[ 0, \frac{1}{2} \right] \),
   b) \( \left[ \frac{1}{2}, 2 \right] \),
   c) \( [2, \infty) \).