1. Consider the function \( f(x, y) = xy(3 - x - y) \). Find the largest value of \( f(x, y) \) over the first quadrant \( x, y \geq 0 \).

2. Evaluate the generalized integral
\[
\iiint_D \frac{x^2 z}{x^2 + y^2 + z^2} e^{-x^2 - y^2 - z^2} \, dx \, dy \, dz,
\]
where \( D : z \geq \sqrt{3(x^2 + y^2)} \).

3. Consider the system of equations
\[
\begin{align*}
x^2 + y^2 - z^2 &= 2 \\
x + y - 2e^z &= 0.
\end{align*}
\]
Verify that \((x, y, z) = (1, 1, 0)\) is a solution. Show that the system can be solved for \( y \) and \( z \) as smooth functions of \( x \) near the point \((1, 1, 0)\) and compute \( y'(1) \) and \( z'(1) \).

4. Find the general \((C^2\)-smooth) solution \( u(x, y) \) to the problem
\[
\cos^2(x)u''_{xy} + u''_{yy} = 0.
\]
It is helpful to introduce the new variables \( s = \tan x, t = y - \tan x \). Also find a particular solution such that \( u(x, 0) = \tan^2 x \) and \( u'_x(x, 0) = 0 \).

5. Find an equation of the curve in the \( xy \)-plane which passes through \((1, 1)\) and which intersects all level curves of the function \( f(x, y) = x^2 e^y \) orthogonally.

6. Let \( u(x, y) \) be a \( C^\infty \)-smooth function. Given \( \epsilon > 0 \) let \( D_\epsilon : x^2 + y^2 < \epsilon^2 \) be the \( \epsilon \)-neighbourhood of the origin. Denote the average value of \( u \) over \( D_\epsilon \) by
\[
A(\epsilon, u) = \frac{1}{\pi \epsilon^2} \iint_{D_\epsilon} u \, dA.
\]
   a) Define the second-order Taylor polynomial \( P_2(x, y) \) of \( u \) about \((0, 0)\). What does Taylor’s formula say about the size of the difference \( u(x, y) - P_2(x, y) \) as \((x, y) \to (0, 0)\)?
   b) Assume that \( \Delta u(0, 0) > 0 \) where \( \Delta u = u''_{xx} + u''_{yy} \) is the Laplacian. Prove that the inequality \( A(\epsilon, u) > u(0, 0) \) holds for all sufficiently small \( \epsilon > 0 \).

Comment. A function which satisfies \( \Delta u > 0 \) at a point is said to be subharmonic there.