1. Determine the maximum and the minimum of the function \( f(x, y) = x + 8y \) when \( x^4 + y^4 = 17 \).

2. Compute the line-integral
\[
\int_{\gamma} (e^x \cos x - y) \, dx + (2xy + \arctan(y^2)) \, dy
\]
where \( \gamma \) is the positively oriented boundary of the domain \( D : 0 \leq x \leq 1, x^2 \leq y \leq x \).

3. Determine the flux of the vector-field \( u = (xz^2, x^2y - z^3, e^{x^2+y^2} + y^2z) \) outwards through the hemisphere \( x^2 + y^2 + z^2 = a^2, z \geq 0 \), where \( a > 0 \) is a constant. (By "outwards" we mean the normal direction with positive \( z \)-component).

4. Define \( g_n(x) = n^\alpha xe^{-nx} \) where \( \alpha \) is a constant and \( n \) is a positive integer. Determine the limit \( g(x) = \lim_{n \to \infty} g_n(x) \) when \( x \geq 0 \). Determine the values of \( \alpha \) for which the convergence \( g_n \to g \) is uniform on \([0, \infty)\).

5. Find a \( C^1 \)-function \( f(x) \) such that the field
\[
u = (f(x)z \cos y, -f(x)z \sin y, f(x) \cos y)
\]
is conservative and maps \((0, 0, 0)\) to \((0, 0, 1)\). Determine a potential function for the field \( \nu \) corresponding to your choice of \( f \).

6. Let \( u(x, y) \) be a \( C^2 \)-smooth function, which satisfies \( \Delta u \geq 0 \) everywhere on \( \mathbb{R}^2 \), where \( \Delta u = u_{xx} + u_{yy} \) is the Laplacian. (Such functions are called subharmonic). For \( r > 0 \) we let \( \gamma_r : x^2 + y^2 = r^2 \) be the circle about \((0, 0)\) of radius \( r \). Let \( I(r) \) be the average value of \( u \) over \( \gamma_r \), i.e.,
\[
I(r) = \frac{1}{2\pi r} \int_{\gamma_r} u(x, y) \, ds, \quad r > 0.
\]

a) Prove that \( I(r) \to u(0, 0) \) as \( r \to 0 \).

b) Prove that \( I(r) \) is an increasing function of \( r \).