Answers

1. \( x(t) = 2 \cosh(t) - 1. \)

2. \( e^x \) is an integrating factor. The equation has the implicit solution \((x-1)e^x + y^2e^x = C.\)

3. \[
\bar{x}(t) = \begin{pmatrix} -1 - e^{2t}(t - 2) \\ 1 + e^{2t}(t - 1) \end{pmatrix}.
\]

4. a) —
   b) \( x(t) = c_1e^t + c_2t^2e^t. \)
   c) \( x(t) = -\frac{1}{2} \int e^{-t}f(t)\,dt + \frac{t^2 + 1}{2} \int \frac{e^{-t}}{t^2}f(t)\,dt \) (where the indefinite integrals contain two arbitrary constants).

5. \( x_1(t) = \sinh(\sqrt{t}), \) \( x_2(t) = \cosh(\sqrt{t}) \) are two linearly independent Frobenius series solutions.

6. a) —
   b) Any linearly independent set of functions whose Wronskian has a zero will do, since the Wronskian of a fundamental set of solutions of a homogeneous linear equation is non-zero for all \( t. \) Assuming that the interval contains \( t = 0, \) we can e.g. take \( x_1(t) = t, \) \( x_2(t) = t^2, \ldots, \) \( x_n(t) = t^n, \) in which case the Wronskian is zero at \( t = 0 \) (the last column of the determinant only contains zeros).