In order to sit the examination you must be enrolled in the course. No aids. Use the papers provided by the department and write on one side of each sheet only. Fill in the cover completely and write your initials on each sheet. Write legibly. Give concise and short arguments and draw figures when applicable.

1. a) Find all solutions of the Diophantine equation
   \[29x + 11y = 200.\]

   b) Find all solutions \((x, y)\) for which \(x\) and \(y\) are positive integers.

2. Express the complex roots of the equation
   \[z^3 = \frac{(1 + i)^5}{(1 + \sqrt{3}i)^3(1 - i)^2}\]
   in the form \(re^{i\theta}\) and plot them as points in the complex plane.

3. a) Find the constant term in the expansion of
   \[(x^2 - \frac{1}{2x})^9.\]

   b) How many distinct arrangements can be made using all the letters in the word TALLAHASSEE?

4. The equation
   \[z^3 - (1 + 3i)z^2 - (6 - 3i)z + 8 + 6i = 0\]
   has a real root. Find all complex roots of the equation (including the real roots).

5. Prove by induction that
   \[\sum_{k=1}^{n} k^2 \geq \frac{n(n + 1)^2}{4}\]
   for all positive integers \(n\).

6. a) Let \(a\) and \(b\) be positive integers and assume that \(b\) is odd. Show, e.g. using the formula for geometric sums, that \(a^b + 1\) is divisible by \(a + 1\).

   b) Let \(k\) be a positive integer. Show that if \(2^k + 1\) is a prime number, then \(k = 2^n\) for some natural number \(n\).