In order to sit the examination you must be enrolled in the course. No aids. Use the
papers provided by the department and write on one side of each sheet only. Fill in the
cover completely and write your initials on each sheet. Write legibly. Give concise and
short arguments and draw figures when applicable.

1. Show that the vectors \( e_1 = (1, 1, 2), e_2 = (1, 2, 3), \) and \( e_3 = (2, 1, 2) \) form a basis for
two-space. Find the coordinates of the vector \( u = (2, 3, 4) \) with respect to this basis.

2. The line through the point \( P = (3, 4, 3) \) with direction vector \( (1, 1, 1) \) and the line
through the point \( Q = (4, 8, 10) \) with direction vector \( (1, 2, 3) \) intersect in a point \( R \).
(Please provide an orthonormal system.)
   a) Find \( R \).
   b) Compute the area of the triangle \( PQR \).

3. Show that the matrix
\[
A = \begin{bmatrix}
1 & 2 & 3 \\
1 & 1 & -1 \\
1 & 1 & 1
\end{bmatrix}
\]
is invertible and compute the inverse \( A^{-1} \).

4. A plane parallel to the line \( (x, y, z) = (1, 3, 7) + t(1, 2, -1) \) passes through the points
\( (1, 1, 1) \) and \( (1, -2, 2) \). (Please provide an orthonormal system.)
   a) Find an equation of the plane in the form \( Ax + By + Cz + D = 0 \).
   b) Compute the distance from the point \( P = (5, 2, 3) \) to the plane.
   c) Find the point in the plane closest to \( P \).

5. The vertices of a certain tetrahedron are located at the points
\[
P = (-1, 1, -2), \quad Q = (3, -1, 2), \quad R = (6, -1, -1), \quad S = (3, 2, -1)
\]
of which the first three lie in the plane \( x + 4y + z = 1 \). (Please provide an orthonormal sys-
tem.)
   a) Compute the angle between the face \( PQR \) and the edge \( PS \).
   b) Compute the angle between the faces \( PQR \) and \( PQS \).
   c) Compute the volume of the tetrahedron.

6. Let \( C \) be the circle of intersection of the plane \( 2x + y + 2z = 2 \) and the sphere
\( (x + 5)^2 + (y + 5)^2 + (z + 5)^2 = 225 \), and let \( M \) be the plane \( 23x + 19y + 8z = 386 \).
Find the points on \( C \) closest to and farthest from \( M \). (Please provide an orthonormal sys-
tem.)