In order to sit for the examination you must be enrolled in the course. No aids. Use the papers provided by the department and write on one side of each sheet only. Fill in the cover completely and write your initials on each sheet. Write legibly. Give concise and short arguments and draw figures when applicable.

1. Show that the vectors \( u_1 = \frac{1}{\sqrt{2}}(1, 0, 1) \), \( u_2 = \frac{1}{\sqrt{2}}(1, 0, -1) \), and \( u_3 = (0, 1, 0) \) form an orthonormal basis for 3-space. Determine the coordinates of the vector \( v = (3, 3, 1) \) with respect to this basis. (Orthonormal system.)

2. Let \( \ell \) be the line through the points \( P_1 = (2, 4, -5) \) and \( P_2 = (5, 2, -5) \), and let \( M \) be the plane that is parallel to \( \ell \) and passes through the points \( P_3 = (0, 1, -1) \) and \( P_4 = (2, 1, 0) \). (Orthonormal system.)
   a) Determine an equation in normal form for the plane \( M \).
   b) Show that the triangle \( P_1P_2P_3 \) is right-angled.

3. Find the point on the sphere \( (x-1)^2 + (y-2)^2 + (z-3)^2 = 9 \) that is closest to the point \( P = (7, -1, -3) \). Compute the least distance from \( P \) to the sphere \( S \). (Orthonormal system.)

4. Consider the matrices
   \[
   A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 3 & 2 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 3 & 1 \end{bmatrix}.
   \]
   a) Compute the inverse of the matrix \( C = A + E \) where \( E \) denotes the identity matrix.
   b) Solve the matrix equation \( AX + X = B \).

5. Consider the planes with equations
   \[
   x + 2y + 2z + 3 = 0, \quad x - 2y + 2z - 1 = 0 \quad \text{and} \quad 2x + y + 2z + 1 = 0.
   \]
   (Orthonormal system.)
   a) Show that the planes intersect in a point and find that point.
   b) Show that the point \( (1, -1, 7) \) is at equal distance from the three planes.
   c) Find some line \( \ell \) with the property that each point on \( \ell \) is at equal distance from the three planes.

6. Let \( A \) be the centre of mass of the face \( PQR \) of the tetrahedron \( OPQR \). Moreover, let \( B, C, \) and \( D \) be the midpoints of the edges \( OP, OQ, \) and \( OR, \) respectively. Find the volume of the tetrahedron \( ABCD \) in relation to the volume of the tetrahedron \( OPQR \).