In order to sit for the examination you must be enrolled in the course. No aids. Use the papers provided by the department and write on one side of each sheet only. Fill in the cover completely and write your initials on each sheet. Write legibly. Give concise and short arguments and draw figures when applicable.

1. How many nine-digit numbers contain exactly three 0’s and three 1’s and three 2’s? Justify your answer!

2. a) Solve the Diophantine equation

\[ 68x - 26y = 80. \]

b) Find a greatest common divisor of the polynomials

\[ f(x) = x^4 - 4x^3 + 6x^2 - 4x + 5, \quad g(x) = x^4 - 2x^3 + 2x^2 - 10x + 25. \]

3. The equation

\[ z^4 - 4z^3 + 14z^2 - 36z + 45 = 0 \]

has at least one root the real part of which equals 0. Solve the equation completely.

4. State in the form \( a + ib, a, b \in \mathbb{R} \), as well as in polar form the roots of the two equations

a) \( z^2 - (8 - 8i)z - 64i = 0, \)

b) \( z^6 - (8 - 8i)z^3 - 64i = 0. \)

5. Show, by means of induction, that the inequality

\[ \binom{2n}{n} < \frac{4^n}{\sqrt{n+1}} \]

holds for all positive integers \( n \).

6. Let

\[ z = (3 + 7i)^n + (7 + 3i)^n, \quad n \in \mathbb{N}. \]

For which natural numbers \( n \) is the number \( z \) purely imaginary?