Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. Let $\text{SL}(2, \mathbb{R})$ be the set of all $2 \times 2$ matrices
   \[
   \begin{bmatrix}
   a & b \\
   c & d
   \end{bmatrix}
   \]
   such that $a, b, c, d \in \mathbb{R}$ and $ad - bc = 1$. Prove that $\text{SL}(2, \mathbb{R})$ is a group under matrix multiplication.

2. Find the multiplicative inverse of $x^2 + 1$ in $\mathbb{Q}[x]/(x^4 - 2)$.

3. Let $U_{35}$ denote the group of units in $\mathbb{Z}_{35}$. Is there a divisor $d$ of $|U_{35}|$ such that no element in $U_{35}$ has order $d$?

4. Let $K$ be a field such that $\mathbb{Q} \subset K \subset \mathbb{R}$ and $[K : \mathbb{Q}] = 2$. Prove that $K = \mathbb{Q}(\sqrt{d})$ for some integer $d$.

5. Let $n$ be an integer, $n \geq 3$. Prove that every element of $A_n$ is a product of 3-cycles.

6. Let $I$ be the ideal
   \[
   I = \{f(x)(2x - 1) + g(x)(x - 2) \mid f(x), g(x) \in \mathbb{Z}[x]\}
   \]
   in $\mathbb{Z}[x]$. Show that $I \neq \mathbb{Z}[x]$. 