Problem 1. The solution of the recursion problem is:
\[ a_n = -2^n + (-3)^n + n2^n, \quad n = 0, 1, 2, \ldots \]

Problem 2. Encoding function \( E \), code words and weight function \( w \) for our code \( C \) are given by the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>y = E(x)</th>
<th>w(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>00000</td>
<td>0</td>
</tr>
<tr>
<td>001</td>
<td>00101</td>
<td>2</td>
</tr>
<tr>
<td>010</td>
<td>01111</td>
<td>4</td>
</tr>
<tr>
<td>011</td>
<td>01010</td>
<td>2</td>
</tr>
<tr>
<td>100</td>
<td>11010</td>
<td>3</td>
</tr>
<tr>
<td>101</td>
<td>11111</td>
<td>5</td>
</tr>
<tr>
<td>110</td>
<td>10101</td>
<td>3</td>
</tr>
<tr>
<td>111</td>
<td>10000</td>
<td>1</td>
</tr>
</tbody>
</table>

The separation of a linear code equals the minimum weight of its nonzero code words. Our code \( C \) has separation 1. The received word 10101 is a code word in \( C \). Indeed, we have that \( E(110) = 10101 \). The received word 00111 is not a code word in \( C \).

Problem 3. We shall use the so-called principle of inclusion and exclusion. Denote by \( X \) the set of all integers \( n \) such that \( 1 \leq n \leq 143 \). Denote by \( X_2 \) the subset of \( X \) consisting of the even integers in \( X \), denote by \( X_3 \) the subset of integers in \( X \) divisible by 3, and denote by \( X_7 \) the set of integers in \( X \) that are divisible by 7. Phrased in these terms we want to calculate the number of integers in the set \( X \setminus (X_2 \cup X_3 \cup X_7) \).

By the above mentioned principle we have that
\[
|X \setminus (X_2 \cup X_3 \cup X_7)| = |X| - (|X_2| + |X_3| + |X_7|) + (|X_2 \cap X_3| + |X_2 \cap X_7| + |X_3 \cap X_7|) - |X_2 \cap X_3 \cap X_7|,
\]
where the symbol \( |\cdot| \) is used to indicate the number of elements in a set. A calculation gives that \( |X_2| = 71 \), \( |X_3| = 47 \) and \( |X_7| = 20 \). Next observe that the set \( X_2 \cap X_3 \) consists of all integers in \( X \) divisible by 6. Arguing this way we see that \( |X_2 \cap X_3| = 23 \), \( |X_2 \cap X_7| = 10 \), \( |X_3 \cap X_7| = 6 \), and \( |X_2 \cap X_3 \cap X_7| = 3 \). We now have that the number of integers \( 1 \leq n \leq 143 \) not divisible by 2, 3 or 7 equals
\[
|X \setminus (X_2 \cup X_3 \cup X_7)| = 143 - (71 + 47 + 20) + (23 + 10 + 6) - 3 = 41.
\]
**Problem 4.** Recall that equivalence relations correspond to partitions. There is exactly one partition of the set \( S = \{1, 2, 3, 4\} \) having a block with 4 or more elements. The number of equivalence relations on \( S \) such that every equivalence class has at most 3 elements is
\[
\sum_{k=1}^{4} S(4, k) - 1 = 1 + 7 + 6 + 1 - 1 = 14,
\]
where \( S(n, k) \) are Stirling numbers of the second kind. \( \Box \)

**Problem 5.** The solutions of the congruences are the polynomials \( f(x) \) in \( \mathbb{Z}_3[x] \) of the form
\[
f(x) = 2x^3 + 2x + 1 + (x^2 + 1)(x^3 + 2x + 2)g(x), \quad g(x) \in \mathbb{Z}_3[x].
\]

\( \Box \)

**Problem 6.** We first notice that the generating function
\[
f(x) = \sum_{n=0}^{\infty} c_n x^n
\]
for the sequence \( \{c_n\}_{n=0}^{\infty} \) is given by
\[
f(x) = \frac{1}{1 - x} \frac{1}{1 - x^2} = \frac{1}{(1 - x)^2(1 - x^2)} = \frac{1}{(1 - x)^3(1 + x)}.
\]
A calculation gives the partial fraction decomposition that
\[
f(x) = \frac{1}{2} \frac{1}{(1 - x)^3} + \frac{1}{4} \frac{1}{(1 - x)^2} + \frac{1}{8} \frac{1}{1 - x} + \frac{1}{8} \frac{1}{1 + x}.
\]
Using the standard power series expansion
\[
\frac{1}{(1 - x)^k} = \sum_{n=0}^{\infty} \binom{n + k - 1}{n} x^n
\]
we have the expansion
\[
f(x) = \sum_{n=0}^{\infty} \left( \frac{1}{2} \binom{n + 2}{n} + \frac{1}{4} \binom{n + 1}{n} + \frac{1}{8} + \frac{1}{8}(-1)^n \right) x^n
\]
so that
\[
c_n = \frac{1}{2} \binom{n + 2}{n} + \frac{1}{4} \binom{n + 1}{n} + \frac{1}{8} + \frac{1}{8}(-1)^n
\]
for \( n \geq 0 \). A straightforward calculation now gives that
\[
c_n = \frac{1}{4} n^2 + n + \frac{7}{8} + \frac{1}{8}(-1)^n
\]
for \( n \geq 0 \). \( \Box \)