Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. Determine the degree of the field extension
   \[ \mathbb{Q}(\sqrt{2}, \sqrt{3}) \supseteq \mathbb{Q} \]

2. Let \( M \) be the ideal in \( \mathbb{Z}[x] \) generated by 2 and \( x^2 + x + 1 \). Show that \( M \) is a maximal ideal in \( \mathbb{Z}[x] \).

3. Let \( \phi \) be an automorphism of the field \( \mathbb{R} \). Show that
   (i) \( \phi(x) = x \) if \( x \in \mathbb{Q} \),
   (ii) \( \phi(x) \geq 0 \) if \( x \geq 0 \),
   (iii) \( \phi \) is increasing,
   (iv) \( \phi \) is the identity map of \( \mathbb{R} \).

4. Show that there are no simple groups of order 160.

5. Find the Jordan normal form of the matrix
   \[
   A = \begin{bmatrix}
   -1 & 6 & -9 \\
   0 & 2 & 0 \\
   1 & -2 & 5 
   \end{bmatrix}
   \]

6. Show that the presentation
   \[ G = \langle a, b, c \mid a^2 = b^2 = c^3 = 1, ab = ba, cac^{-1} = b, cbc^{-1} = ab \rangle \]
   defines a group \( G \) that is isomorphic to \( A_4 \).