1. Show that \( \frac{d}{dx}(x^2 \sin x^{-3}), x > 0 \), is not in \( L^1(0, 1) \).

2. Let \( \mu \) and \( \nu \) be positive measures on a measurable space \( (X, \mathcal{M}) \).
   (a) Suppose \( \nu \) is absolutely continuous with respect to \( \mu \) with density function \( f \in L^p(\mu) \) for some \( p > 1 \). Show that there exists a constant \( C \) so that \( \nu(E) \leq C(\mu(E))^{(p-1)/p} \) for any \( E \in \mathcal{M} \).
   (b) Show that if the inequality \( \nu(E) \leq C\mu(E) \) holds for a fixed constant \( C \) and all \( E \in \mathcal{M} \), then \( \nu \) is absolutely continuous with respect to \( \mu \) with Radon-Nikodym derivative \( f \) satisfying \( 0 \leq f \leq C \) a.e. \( (\mu) \).

3. Determine \( \lim_{n \to \infty} \int_{-\infty}^{\infty} \frac{dx}{n(e^{x^2} - 1) + 1/n} \).

   Hint: Note that \( e^t \geq 1 + t \).

4. Suppose \( f \) is measurable on the interval \((0, 1)\) and assume that
   \[
   \int_0^1 x^{-2} |f(x)| \, dx < \infty.
   \]
   Let \( \{a_k\}_{k=1}^\infty \) be a real sequence with \( 0 < a_k \leq 1 \) and \( \sum_{k=1}^\infty a_k < \infty \). Show that the series \( \sum_{k=1}^\infty f(a_k x) \) converges a.e. in \((0, 1)\).

5. Suppose \( g \geq 0 \) is bounded and Lebesgue measurable in \( \mathbb{R}^n \), that \( g \equiv 0 \) outside \( B(0, 1), \) and that \( \int_{\mathbb{R}^n} g = 1 \). Put \( g_\varepsilon(x) = \varepsilon^{-n} g(x/\varepsilon), \varepsilon > 0 \). Show that if \( f \) is integrable on every compact set in \( \mathbb{R}^n \), then
   \[
   \int_{\mathbb{R}^n} g_\varepsilon(y)f(x-y) \, dy \to f(x) \quad \text{as} \quad \varepsilon \to 0
   \]
   for almost all \( x \in \mathbb{R}^n \).