1. Determine if the following congruences are solvable or not:

\[
x^2 + 5x - 7 \equiv 0 \pmod{337}
\]

\[
x^2 + 5x - 7 \equiv 0 \pmod{347}
\]

(337 and 347 are primes.)

2. Let the integer \( n > 1 \) have the prime factorization

\[ n = p_1^{k_1} \cdots p_r^{k_r}. \]

Prove that

\[ \sum_{d|n} \mu(d)\sigma(d) = (-1)^r p_1 \cdots p_r. \]

3. Determine if

\[ 8x^2 + 30y^2 - 15z^2 = 0 \]

has a solution \( x, y, z \in \mathbb{Z} \), not all zero.

4. Without actually finding them, determine the number of solutions of the congruence

\[ x^2 \equiv 3 \pmod{11^2 \cdot 23^2}. \]

5. Prove that of four consecutive integers, at least one is not representable as a sum of two squares.

6. Determine all solutions to

\[ x^3 + 4x - 2 \equiv 0 \pmod{7^3}. \]