Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. Find the values of the Legendre symbols 
   \[
   \left( \frac{82}{223} \right), \quad \left( \frac{82}{257} \right).
   \]

2. Let \((u_n)_{n=1}^{\infty}\) be the sequence of Fibonacci numbers, determined by 
   \[u_{n+1} = u_n + u_{n-1}, \quad u_1 = u_2 = 1.\]
   Prove that 
   \[2^{n-1}u_n \equiv n \pmod{5},\]
   for all \(n \geq 1\).

3. Show that \(3|\sigma(3n+2)\) for each integer \(n \geq 0\), where \(\sigma(k)\), denotes the sum of the positive divisors of \(k\).

4. Let \(n = x^2 + y^2\) where \(x, y \in \mathbb{Z}\), be of one of the forms \(n = 2^{2k}m\) respectively \(n = 2^{2k+1}m\) with \(m\) odd. Prove that \(2^k\) divides both \(x\) and \(y\).

5. Let \(p\) be an odd prime, and let \(r\) be a primitive root \(\pmod{p}\) such that \(r^{p-1} \neq 1 \pmod{p^2}\). Show that \(r\) is a primitive root \(\pmod{p^k}\) for all \(k \geq 1\).

6. Let \(x\) be an irrational real number and let \((p_n/q_n)_{n=0}^{\infty}\) be the convergents to \(x\). Show that at least one of two consecutive convergents \(p_n/q_n\), \(p_{n+1}/q_{n+1}\) satisfy
   \[\left| x - \frac{p}{q} \right| < \frac{1}{2q^2}.\]