Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. Determine the last two decimal digits in
   \[2^{23}.\]

2. Let \(p\) denote a prime, \(p > 3\). Determine a congruence criterion on \(p\), for when the congruence
   \[x^2 - 5x + 7 \equiv 0 \pmod{p}\]
   is solvable.

3. Let \((u_n)_{n=1}^\infty\) denote the sequence of Fibonacci numbers defined by
   \[u_{n+2} = u_{n+1} + u_n, \quad u_1 = u_2 = 1.\]
   Give a congruence criterion on those integers \(n\) for which
   \[11 | u_n.\]

4. Let \(x, y, n, p\) be positive integers such that
   \[x^2 + ny^2 = p,\]
   where \(p\) is a prime such that \(p \neq n\). Show that the Legendre symbol \((-n/p) = 1.\)

5. Let \(q, p\) be primes such that \(q = 4p + 1\). Show that \(-2\) is a primitive root \(\pmod{q}\).

6. (i) Determine the continued fraction expansion of \(\sqrt{3}\). \(1p\)
   (ii) Let \(p_n/q_n\) be the convergents to \(\sqrt{3}\). Show that
   \[\left| \sqrt{3} - \frac{p_{2n+1}}{q_{2n+1}} \right| < \frac{1}{2\sqrt{3}q_{2n+1}^2}. \quad 4p\]