Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. Determine, using Legendre symbols, whether or not the following congruences are solvable:

\[ x^2 + 3x + 5 \equiv 0 \pmod{233}, \]
\[ x^2 + 4x + 7 \equiv 0 \pmod{239}. \]

The numbers 233, 239 are primes.

2. Let \( d = a^2 + 1 \), where \( a \) is a positive integer. Show that the simple continued fraction of \( \sqrt{d} \) has period of length 1.

3. Show that if \( p \) is a prime and \( p = 2q + 1 \), where \( q \) is an odd prime and \( a \) is a positive integer with \( 1 < a < p - 1 \), then \( p - a^2 \) is a primitive root modulo \( p \).

4. Find integers \( a, b, c \) such that

\[ u_{4n} \equiv an \pmod{5}, \]
\[ u_{4n+2} \equiv bn + c \pmod{5}. \]

Verify the congruences obtained. Here \( (u_n) \) denotes the sequence of Fibonacci numbers starting with \( u_1 = u_2 = 1 \).

5. Show that

\[ \sum_{1 \leq n \leq x} \mu(n) \left\lfloor \frac{x}{n} \right\rfloor = 1 \]

for all real \( x \geq 1 \). Here \( [t] \) denotes the greatest integer that is \( \leq t \).

6. Show that a prime \( p \) can be represented

\[ p = 3x^2 - y^2 \]

with \( x, y \in \mathbb{Z} \) if and only if \( p = 2, p = 3 \) or \( p \equiv 11 \pmod{12} \).