Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. (a) Show that 2 is a primitive root modulo 19.
   (b) Calculate a table of indices with respect to the primitive root 2.
   (c) Find all solutions to the congruence
   \[ 6x^{15} \equiv 13 \pmod{19}. \]

2. Show that \( \varphi(n) \equiv 2 \pmod{4} \) if and only if
   \[ n = p^a \text{ or } n = 2p^a, \text{ or } n = 4, \]
   where \( p \) in both cases is a prime \( \equiv 3 \pmod{4} \) and \( a > 0 \). (Here \( \varphi \) denotes Euler’s \( \varphi \)-function.)

3. For any positive integer \( n \), prove that
   \[ \sum_{d \mid n} \sigma(d) = \sum_{d \mid n} \frac{n}{d} \tau(d). \]
   (\( \sigma \) = sum of the positive divisors, \( \tau \) = number of positive divisors)

4. Let \( p \) and \( q \) be primes such that \( p \) is of the form \( 4k + 3 \), and \( q = 2p + 1 \). Show that \( q \) divides \( 2^p - 1 \).

5. Let \( p \) be an odd prime, set \( g = \lceil \sqrt{p} \rceil \), and let \( a \) be a quadratic residue modulo \( p \). Show that at least one of the numbers
   \[ 1^2, 2^2, \ldots, g^2 \]
   is congruent modulo \( p \) to one of the numbers
   \[ a \cdot 1^2, a \cdot 2^2, \ldots, a \cdot g^2. \]

6. (a) Determine the simple infinite continued fraction expansion of \( \sqrt{34} \).
   (b) Show that there are infinitely many \( (a, b) \in \mathbb{Z}^2, b \geq 1 \), such that
   \[ \left| \sqrt{34} - \frac{a}{b} \right| < \frac{1}{10b^2}. \]