Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. Calculate the Legendre symbols \((232/373)\) and \((295/347)\).

(Here 373 and 347 are primes.)

2. Find all primes \(p\) such that \(a^{25} \equiv a \pmod{p}\) holds for all \(a \in \mathbb{Z}\).

3. Show that for a prime \(p > 3\),

\[
\sum_{\substack{(a/p) = 1, \\ 1 \leq a \leq p-1}} a \equiv 0 \pmod{p}.
\]

4. Let \(a \in \mathbb{Z}\), and let \(n\) be a value assumed by the polynomial \(x^2 - ay^2\) for some \(x, y \in \mathbb{Z}\). Show that for every prime divisor \(p\) of \(n\), either \(p \mid x\) or \((a/p) = 1\).

5. Let \(\alpha\) be an irrational number with simple continued fraction expansion \(\alpha = [a_0; a_1, a_2, \ldots]\).

Show that the simple continued fraction expansion of \(-\alpha\) is

\([-a_0 - 1; 1, a_1 - 1, a_2, a_3, \ldots] \quad \text{if} \quad a_1 > 1, \]

\([-a_0 - 1; a_1 + 1, a_3, a_4, \ldots] \quad \text{if} \quad a_1 = 1.\)

6. For \(x \in \mathbb{R}, x > 0\) and \(n\) a positive integer, let \(f(x, n)\) be the number of positive integers \(k\), such that \(k \leq x\) and \(\gcd(k, n) = 1\). Show that

\[
(i) \quad \sum_{d \mid n} f \left( \frac{x}{d} \cdot \frac{n}{d} \right) = [x],
\]

\[
(ii) \quad f(x, n) = \sum_{d \mid n} \mu(d) \left\lfloor \frac{x}{d} \right\rfloor.
\]

(Here \([x]\) denotes the greatest integer \(\leq x\).)