1. a) Denna jämnar kan man samla 12 cirklar och 3 divider och betyder att (15) = 455.

b) Genom ett lätt att x1 - 5 och x2 = x1 + 1 för 2 ≤ i ≤ 4 vi har den äkvivalent problemet av att hitta antalet av varianter till x1 + x2 + y3 + y4 = 10. Genom att vi hittar att detta jämnar (13) = 286.

c) Med regler om inklusion/-utelämnande med grundvarvet bestående av alla arrangemang avläggare i COOKBOOKS, och förutsättningar

\[ f(x) = (1 + x^2 + x^4 + x^6 + \cdots)(1 + x^2 + x^3 + \cdots)^3 = (1 + x^2 + x^4 + x^6 + \cdots) \sum_{j=0}^{\infty} \binom{3}{j} (-x)^j \]

Hence the sought after coefficient equals \( \sum_{k=0}^{6} \binom{3}{k} (-1)^{2k} = \sum_{k=0}^{6} \binom{2k+2}{2} = \frac{\binom{2}{2} + \binom{4}{2} + \binom{6}{2} + \binom{8}{2} + \binom{10}{2} + \binom{12}{2} + \binom{14}{2}}{2} = \frac{1+6+15+21+15+6+1}{2} = 252 \)

2. Använd prinsippen om inklusion/-utelämnande med grundvarvet bestående av alla arrangemang av läggare i COOKBOOKS, och förutsättningar

\[ N(c_1c_2c_3) = S_0 - S_1 + S_2 - S_3 \] where \( S_0 = \frac{9!}{4!3!} = 7560 \), the size of the grund set. Further \( S_1 = N(c_1) + N(c_2) + N(c_3) = \frac{6!}{3!2!} + \frac{6!}{3!2!} = 360 + 360 + 120 = 840 \). (For example, to obtain \( N(c_2) \) count the arrangements of BOOK, C, O, O, K, S regarding BOOK as one unit.) Next we find that \( S_2 = N(c_1c_2) + N(c_1c_3) + N(c_2c_3) = 3! + 0 + 3! = 12 \) since conditions one and three are incompatible. Finally \( S_3 = N(c_1c_2c_3) = 0 \) and we get our result

\[ N(c_1c_2c_3) = S_0 - S_1 + S_2 - S_3 = 7560 - 840 + 12 - 0 = 6732. \]

3. a) \( s_{n+1} - s_n = \binom{n+1}{2} = \frac{n^2}{2} + \frac{n}{2} \) with \( s_2 = 1 \)

b) The homogenous equations is solved by any constant so we have \( s_n^{(h)} = A \).

To find a particular solution we substitute \( s_n^{(p)} = Bn^3 + Cn^2 + Dn \) into the equation and find by equating coefficients that \( B = 1/6, C = 0, \) and \( D = -1/6 \). Then all solutions are given by \( s_n = s_n^{(h)} + s_n^{(p)} = A + \frac{n^3}{6} - \frac{n}{6} \).

Using our initial condition we find that \( A = 0 \) and hence that \( s_n = \frac{n^3}{6} - \frac{n}{6} \).

4. We first note that 5, 6 and 7 are relatively prime and therefore the Chinese remainder theorem applies if we can transform the equations to the form \( x \equiv a_i \) modulo \( n_i \). For example in \( \mathbb{Z}_5 \) we have that \( 3x + 2 = 1 \Leftrightarrow 3x = 4 \Leftrightarrow x = 3 \) where the second step comes from multiplying with \( 3^{-1} = 2 \). Doing this for the other two equations we get the equivalent system

\[ \begin{cases} x \equiv 3 \pmod{5} \\ x \equiv 3 \pmod{6} \\ x \equiv 2 \pmod{7}. \end{cases} \]
Using the method from the proof of the Chinese remainder theorem we get a solution $x = a_1 s_1 N_1 + a_2 s_2 N_2 + a_3 s_3 N_3 = 3 \cdot (-2) \cdot 42 + 3 \cdot (-1) \cdot 35 + 2 \cdot (-3) \cdot 30 = -252 - 105 - 180 \equiv 93 \pmod{210}$. By the Chinese remainder theorem we know that all solutions are given by $x = 93 + 210n$ where $n$ is any integer.

5. a) A polynomial $h(x) = x^3 + ax^2 + bx + c$ in $\mathbb{Z}_3[x]$ is irreducible if and only if it has no zeroes, that is if

$$
\begin{align*}
  h(0) &= c 
eq 0 \\
  h(1) &= 1 + a + b + c 
eq 0 \\
  h(-1) &= -1 + a - b + c 
eq 0.
\end{align*}
$$

It is easy to find solutions $(a, b, c)$ and hence get irreducible polynomials. Two examples are $(a, b, c) = (1, 0, 2)$ and $(a, b, c) = (2, 0, 1)$ resulting in the polynomials $h_1(x) = x^3 + x^2 + 2$ and $h_2(x) = x^3 + 2x^2 + 1$. To see how many solutions there are look at the system:

$$
\begin{align*}
  h(0) &= c = d \\
  h(1) &= 1 + a + b + c = e \\
  h(-1) &= -1 + a - b + c = f.
\end{align*}
$$

Here there are $2^3 = 8$ choices of non-zero parameters $(d, e, f)$ and for given $(d, e, f)$ the system has a unique solution. (We can see this for example by computing the determinant.) Hence there are 8 irreducible polynomials of degree three.

b) Let us use $h(x) = h_1(x) = x^3 + x^2 + 2$. Then the elements of the field are all equivalency classes $[ax^2 + bx + c]$ where $a, b, c \in \mathbb{Z}_3$ addition and multiplication is done by adding or multiplying representatives of the classes and then finding a representative of degree at most two by taking the remainder after division by $h$. In the given examples $[x^2 + 2x] + [x + 1] = [x^2 + 3x + 1] = [x^2 + 1]$, $[x^2 + x] \cdot [x^2 + 2] = [x^4 + x^3 + 2x^2 + 2x] = [x(x^3 + x^2 + 2) + 2x^2] = [2x^2]$. Inverses can be found using the Euclidean algorithm. By division $x^3 + x^2 + 2 = x^2(x + 1) + 2$. Rearranging and multiplying by 2 we find that $1 = 2(x^3 + x^2 + 2) + x^2(-2x - 2)$ which shows that $[-2x - 2] = [x + 1]$ is an inverse of $[x^2]$.

6. a) The code is an 8-dimensional subspace and therefore contains $11^8$ words.

b) Looking at the columns of $H$ we can see that any two of them are independent but for example the first three columns are linearly dependent. This shows that the separation is three.

c) The syndromes of the words are $(0, 0)$, $(4, 0)$ and $(4, 9)$ respectively. This shows that $w_1$ is a code word and the other two are not. Moreover $w_2$ is not a multiple of a column in $H$ and therefore it must contain at least two errors. Since there are several ways to write $(4, 0)$ as a linear combination of two
columns we cannot properly correct $w_2$. Assume $(4\, 9)$ is a multiple of column $k$. Then $(4\, 9) = a(1\, k)$ and hence $a = 4$ and $k = 4^{-1} \cdot 9 = 3 \cdot 9 = 27 = 5$ in $\mathbb{Z}_{11}$. This shows that $w_3$ can be corrected by subtracting 4 from the element in position 5. This results in the corrected word $w_3 = (1\, 0\, 0\, 1\, 0\, 1\, 0\, 4\, 4)$