1. Find all equilibria for the system
\[
\begin{cases}
x' = y^2 + 2xy, \\
y' = x^2 + xy - 5y - 9,
\end{cases}
\]
and determine their stability properties!

2. The system
\[
\begin{cases}
x' = y^4 - y, \\
y' = x^5 - y^3,
\end{cases}
\]
has the equilibrium (0, 0). Investigate its stability properties.

3. Solve the initial value problem \( x' = Ax, \ x(0) = x_0 \), where
\[
A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \end{pmatrix}
\]
and \( x_0 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \).

4. Show that the system
\[
\begin{cases}
x' = 2y + x \cos(x^2 + y^2), \\
y' = -x + y \cos(x^2 + y^2),
\end{cases}
\]
has infinitely many periodic solutions.

\[ \text{Hint: } -(x^2 + y^2) \leq 2xy \leq x^2 + y^2. \]

5. Suppose \( f(x, y) \geq g(x^2 + y^2) > 0 \) for \( x, y \in \mathbb{R}^2 \), where \( A = \int_1^\infty \frac{ds}{sg(s)} < \infty \). Show that any solution with \( (x(0), y(0)) \neq (0, 0) \) of
\[
\begin{cases}
x' = y + xf(x, y), \\
y' = -x + yf(x, y)
\end{cases}
\]
blows up in finite time, i.e., there is a \( T > 0 \) such that \( (x(t), y(t)) \rightarrow \infty \) as \( t \rightarrow T \). Estimate \( T \) in terms of \( A \) and the initial value of the solution.