1. Find all fixed points of the system
\[
\begin{align*}
x' &= x^2 + xy \\
y' &= x^2 + 2xy + x + 2y + 2.
\end{align*}
\]
Determine their stability properties.

2. The system
\[
\begin{align*}
x' &= -y \cos x \\
y' &= x^3 \cos x - y^3 e^y
\end{align*}
\]
has the fixed point \((0, 0)\). Show that it is asymptotically stable.

3. Solve the problem
\[
\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix}, \quad \begin{pmatrix} x \\ y \end{pmatrix}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.
\]

4. Show that the equation \(z'' - z'(1 - e^{2z^2 + 3(z')^2 - 1}) + z = 0\) has a non-constant, periodic solution.

5. Consider the system
\[
x'(t) = A(t)x(t) + g(t)
\]
where \(A\) and \(g\) are continuous and \(T\)-periodic, that is, \(A(t+T) = A(t)\) and \(g(t+T) = g(t)\) for all \(t\), where \(T > 0\).

a) Show that a solution \(x\) is \(T\)-periodic if and only if \(x(T) = x(0)\).

b) Show that there is a unique \(T\)-periodic solution for every \(T\)-periodic function \(g\) if and only if \(1\) is not an eigenvalue of the monodromy matrix \(\Pi(T, 0)\), where \(\Pi(t, t_0)\) is the principal matrix solution.

Hint: Use the variation of constants formula 
\[
x(t) = \Pi(t, t_0)x(t_0) + \int_{t_0}^{t} \Pi(t, s)g(s) \, ds.
\]