No aids allowed. Use the distributed paper sheets and write only on one side. Fill in the cover sheet completely and write your initials on each sheet. Write legibly (in Swedish or English). Motivate your conclusions clearly and concisely; draw a picture if appropriate.

Test results: Posted Monday, August 26, before 17.00.

Oral exams: August 28–30. State your preference (day and AM/PM) on the cover sheet of your test – at least two options.

1. The origin is a fixed point of the planar autonomous system
   \[
   \begin{align*}
   x' &= 3x^4 + 2axy + \tan(ay), \\
   y' &= ax - \sin(2x) - 2y,
   \end{align*}
   \]
   in which \( a \) is a real parameter. For which values of \( a \) can the linearisation be used to determine the stability properties of the origin? What are then the stability properties?

2. Let
   \[
   A = \begin{pmatrix}
   -2 & 1 & -2 \\
   1 & -2 & 2 \\
   3 & -3 & 5
   \end{pmatrix}.
   \]
   Compute \( e^{tA} \) and describe the general solution of the system \( x' = Ax \).

3. Show that the origin is an asymptotically stable equilibrium point for the system
   \[
   \begin{align*}
   x' &= 2x^2y, \\
   y' &= -x^5 - y^5.
   \end{align*}
   \]

4. Show that the equation \( x'' - x'(1 - 7(x')^4 - 5x^4) + x = 0 \) has a non-trivial periodic solution.

5. Consider the system
   \[
   \begin{align*}
   x' &= x - y - x(x^2 + y^2) + \frac{xy}{\sqrt{x^2 + y^2}}, \\
   y' &= x + y - y(x^2 + y^2) - \frac{x^2}{\sqrt{x^2 + y^2}}.
   \end{align*}
   \]
   Show that \( x_0 = (1, 0) \) is an unstable equilibrium point, but that any solution not starting at the origin converges to \( x_0 \) as \( t \to \infty \).
   
   Hint: use polar coordinates.