1. Find all fixed points of the system
\[
\begin{align*}
    x' &= 2 - y - x^2 \\
    y' &= 2x(x - y).
\end{align*}
\]
Determine whether they are stable, asymptotically stable or unstable.

2. Consider the initial value problem
\[
x' = 1 + \frac{x^2}{1 + t^2}, \quad x(0) = 0.
\]

   a) Show that the function \( f(t, x) = 1 + \frac{x^2}{1 + t^2} \) is Lipschitz continuous with respect to \( x \) on the set \([0, 1] \times [-1, 1]\) and determine a Lipschitz constant.
   
   b) Show that there is a unique solution on the interval \([0, \frac{1}{2}]\), which satisfies \( 0 \leq x(t) \leq 1 \).

3. Prove that the origin is an asymptotically stable fixed point for the system corresponding to the equation
\[
x'' + (x')^3 + x^3 = 0.
\]

4. Show that the boundary value problem
\[
\begin{align*}
    y''(x) &= f(x), & 0 < x < 1, \\
    y(0) &= 1, \\
    y'(1) &= 2
\end{align*}
\]
has a unique solution for each \( f \in C[0, 1] \). Express the solution using Green’s function.

*Please, turn over!*
5. Consider the Sturm-Liouville eigenvalue problem

\[-y'' = \lambda y,\]
\[y(0) = 0,\]
\[y'(1) - y(1) = 0.\]

Let \(\lambda_0 < \lambda_1 < \lambda_2 < \cdots\) be the eigenvalues, ordered increasingly.

a) Show that \(\lambda_0 = 0\) and that \((n\pi)^2 < \lambda_n < \left((n + \frac{1}{2})\pi\right)^2\), \(n \geq 1\).

b) Show that

\[
\lim_{n \to \infty} \left(\lambda_n - \left((n + \frac{1}{2})\pi\right)^2\right) = -2.
\]