1. Find all fixed points of the system
\[
\begin{align*}
    x' &= (x^2 - y)(y^2 - 1) \\
    y' &= x^3 - xy + y^2 - 1
\end{align*}
\]
and determine their stability properties.

2. Prove that the origin is an asymptotically stable fixed point for the autonomous system corresponding to the equation \(x'' + (x')^3 + x^3 = 0\).

3. For which \(b > 0\) does the problem
\[
\begin{align*}
    y'' - 2y' + 2y &= f, & 0 < x < b \\
    y(0) &= y(b) = 0
\end{align*}
\]
have a unique solution for all \(f \in C[0, b]\)? Express the solution using Green’s function when it is possible.

4. Prove that the system
\[
\begin{align*}
    x' &= x + y - x^3 \\
    y' &= -x + y - y^3
\end{align*}
\]
has a non-constant, periodic solution.

5. Consider the equation
\[-u'' + q(x)u = 0, \quad x > 0,
\]
where \(q \in C([0, \infty), \mathbb{R})\). Assume that \(q(x) \to q_\infty\) as \(x \to \infty\) and let \(u\) be a non-trivial solution.

a) Show that \(u\) has infinitely many zeros if \(q_\infty < 0\) and at most finitely many zeros if \(q_\infty > 0\).

b) Show that both alternatives are possible if \(q_\infty = 0\).