1. Find all fixed points of the system
\[
\begin{cases}
  x' = x - y^2 - z^2, \\
  y' = 2x^2 - xy^2 - 1, \\
  z' = -z.
\end{cases}
\]
Determine whether they are stable, asymptotically stable or unstable.

2. Show that the boundary value problem
\[
y''(x) + 9y(x) = f(x), \quad 0 < x < \pi, \\
y(0) = 1, \\
y'(\pi) = -1,
\]
has a unique solution for all \( f \in C[0, \pi] \). Express the solution using Green’s function.

3. Show that the system
\[
\begin{cases}
  x' = 3x + y - 2xe^{x^2+2y^2} \\
  y' = -x + 3y - 2ye^{x^2+2y^2}
\end{cases}
\]
has a regular periodic orbit.

4. Consider the system
\[
\begin{cases}
  x' = x(1 - x - y), \\
  y' = 2 - ye^x.
\end{cases}
\]
Sketch the phase portrait for \( x, y \geq 0 \). Then prove that any solution \((x(t), y(t))\) with \(x(0) \geq 0\) and \(y(0) \geq 0\) remains in the closed first quadrant for \( t \geq 0 \) and converges to the fixed point \((0, 2)\) as \( t \to \infty \).

Please, turn over!
5. Consider the initial value problem

\[ x'(t) = t + e^{-t}e^x, \quad x(0) = 0. \]

Show that solution blows up in finite time, that is, there exists a finite \( T > 0 \) such that \( x(t) \) is defined on the interval \([0, T)\) and \( x(t) \to \infty \) as \( t \to T^- \).

Hint: It might be useful to first show that \( x(t) > t \) for \( t > 0 \).