1. Which of the following statements are correct? Provide a short explanation.
   a) For the Newton method to converge in one step when applied to 
      \( f(x) = x^T H x + c^T x + d \) the matrix \( H \) must be positive-definite. \( \text{(0.2)} \)
   b) Approximation points from the penalty function method are not feasible. \( \text{(0.2)} \)
   c) The Dichotomous search method always converges to the global minimum 
      for a unimodal function. \( \text{(0.2)} \)
   d) The global minimum always exists for a convex function. \( \text{(0.2)} \)
   e) The Steepest Descent method converges fast for quadratic functions. \( \text{(0.2)} \)

2. Consider the function \( f(x, y) = x^4 - 12xy + y^4 \) on \( S = \{(x, y): x \geq 0, y \geq 0\} \).
   a) Find the largest possible (convex) set \( D_{\text{max}} \subset S \) such that \( f(x, y) \) is 
      convex on \( D_{\text{max}} \). \( \text{(0.5)} \)
   b) Show that the minimum of \( f \) over \( D_{\text{max}} \) exists and calculate it. \( \text{(0.5)} \)

3. Consider the LP problem
   \[
   \begin{align*}
   \text{max } (4x_1 + 2x_2 + 3x_3) \quad \text{subject to} \quad & 5x_1 + x_2 + 4x_3 \leq 2, \\
   & x_1 + 2x_2 - x_3 \leq 6, \\
   & 2x_1 + x_2 + x_3 \geq 1, \\
   & x_1, x_2, x_3 \geq 0.
   \end{align*}
   \]
   a) Guess a starting point\(^1\) and solve the problem by the Simplex method. \( \text{(0.5)} \)
   b) State the dual problem and solve it by the CSP\(^2\) using the solution in 3a). \( \text{(0.5)} \)

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\(^1\) without Phase 1  
\(^2\) Complementary Slackness Principle
4. a) Prove that an affine function \( h: \mathbb{R}^n \to \mathbb{R} \) is convex. (0.2)

b) Which of the following sets are convex? (Prove or disprove.) (0.4)

- \( \{(x, y, z) : x \geq y^2 + z^2, z > 0\} \),
- \( \{(x, y, z) : x^2 \geq y^2 + z^2, y > 0\} \),
- \( \{(x, y, z) : x^2 \geq y^2 + z^2, x > 0\} \).

c) Prove that the following function is convex in \( \mathbb{R}^3 \) (0.4)

\[
f(x, y, z) = \sqrt{1 + x^2 + y^2 + z^2}.
\]

5. Solve the optimization problem below using KKT conditions

\[
\min (3x + 4y) \quad \text{subject to} \quad 2y \leq x^2 + y^2 \leq 4, \ x \geq 0.
\]

6. a) Consider the problem in 5 and set \( X = \{(x, y) : x \geq 0\} \). Calculate the dual function and make sure that there is no duality gap here. (You may use some calculations from Problem 5). (0.6)

b) In the course, a sufficient condition for a KKT point to be a global minimizer of a constrained problem is given. State the condition as a theorem and prove it. (0.4)

**GOOD LUCK!**