Answers and Comments

1. a) No, since the dual system (***) on page 135 has a solution \( y = (2, 1, 3)^T \).
   
   b) See the proof to Lemma 1, p. 43.

2. a) Vector \( a \) is the Newton direction (it minimizes the quadratic form in one step.)
   Vector \( b \) is none (it is a tangent vector). Vector \( c \) is none (it could be the Steepest descent direction, but it has the wrong sign).
   
   b) Vectors \( a \) and \( b \), since \( a = -x_0 \), \( c \parallel \nabla f(x_0) = 2Hx_0 \) and \( b^T c = 0 \Rightarrow b^T H a = 0 \).

3. a) Pick \( (s_1, x_1, x_2) \) as a BFS, \( \min = 1 \) at \( x_{opt} = (0, 1, 1) \).
   
   b) The dual problem has the solution \( y_{opt} = (0, 1, 0) \), \( \max = 1 \).

4. a) Not convex (e.g. draw the set profile for \( y = 0 \)).
   
   b) Convex, since \( f = x^2 + y^2 - z \) and \( g = x^2 + y^2 + z \) are both convex \( \Rightarrow h = \max\{f, g\} \)
   \( \Rightarrow \{h \leq 1\} \) is a convex set.
   
   c) Only for \( \alpha = 0 \) (e.g. study the Hessian by (modified) Sylvester criterion for \( \alpha \neq 0, 1 \) and study cases \( \alpha = 0 \) and \( \alpha = 1 \) separately).

5. The set is compact \( \Rightarrow \min \) exists. No CQ points. KKT points: \( (0, 0, 0) \) with \( f = 0 \),
\( \pm(1, -1, 0) \) with \( f = -2 \) and \( \pm(1, 1, -4) \) with \( f = -14 \). The last two are the minimum points.

6. a) The Lagrange function \( L \) is a quadratic function with the indefinite Hessian \( \Rightarrow \Theta(u, v) = -\infty \) for all \( u \geq 0 \) and \( v \). Maximization gives again \( -\infty \). The obvious duality gap makes it impossible to use the dual problem in order to solve the primal one.
   
   b) To minimize \( f \) is the same as to minimize \( g = f^2 \). The stationary point equation \( \nabla g = 0 \) is equivalent to the normal equation \( \Rightarrow \) the solution to the normal equation is the stationary point + \( g \) is convex \( \Rightarrow \) it is the global minimum.

\(^1\)For re-exams only answers are provided.