Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. Let \( p(t) = t^4 + 3t^2 - t + 2 \in \mathbb{Q}[t] \). Show that \( p \) is irreducible over \( \mathbb{Q} \).

2. Give an example of an extension \( L : K \) such that \( \text{char}(K) = p > 0 \), \( L : K \) is finite but not normal.

3. Let \( \mathbb{C}(t) \) be the field of rational expressions in \( t \), and let

\[
z = \frac{t^2 + at + b}{t^2 + ct + d} \in \mathbb{C}(t)
\]

where \( t^2 + at + b \) and \( t^2 + ct + d \) are relatively prime. Find the degree

\[
[\mathbb{C}(t) : \mathbb{C}(z)].
\]

4. Let

\[
p(t) = t^4 + 2t^2 - 2 \in \mathbb{Q}[t],
\]

and let \( L : \mathbb{Q} \) be a splitting field of \( p(t) \).

(i) Determine the Galois group \( \Gamma(L : \mathbb{Q}) \). (3p)

(ii) Determine all intermediate fields \( K, \mathbb{Q} \subset K \subset L \) such that \( [K : \mathbb{Q}] = 2 \). (2p)

5. Determine and prove a formula for the number of monic irreducible polynomials of degree \( d \) over \( \mathbb{Z}/p\mathbb{Z} \), \( p \) a prime.