Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. Determine whether or not $\mathbb{Q}(\sqrt{2}, \sqrt{5})$ is a simple extension.

2. Show that

$$t^4 - t^2 + 3t + 2$$

is irreducible over $\mathbb{Q}$.

3. Let $E$ be a splitting field of $t^3 - 2$ over $\mathbb{Q}$.
   (i) Describe the action of the Galois group $G = \Gamma(E : \mathbb{Q})$. (1p)
   (ii) Calculate the subgroups of $G$. (2p)
   (iii) Describe the intermediate fields of $E : \mathbb{Q}$. (2p)

4. Let $L : K$ be a radical extension in $\mathbb{C}$ and let $M$ be an intermediate field. Show that $M : K$ need not be radical.

5. Construct an extension $E : \mathbb{Q}$ such that the Galois group $\Gamma(E : \mathbb{Q})$ is cyclic of order 5.