Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

All Lie algebras below are assumed to be finite dimensional over an algebraically closed field $F$ of characteristic 0.

1. Let $L$ be a solvable Lie algebra with $\dim L > 1$. Show that if $I$ is a minimal nonzero proper ideal in $L$, then $\dim I = 1$.

2. Let $L$ be a semi-simple Lie algebra, $H$ a maximal toral subalgebra, $\Phi^+$ a system of positive roots of $(L, H)$.
   (i) Show that 
   $$B = H \oplus \bigoplus_{\alpha \in \Phi^+} L_{\alpha}$$
   is a maximal solvable subalgebra of $L$.
   (ii) Show that 
   $$N = \bigoplus_{\alpha \in \Phi^+} L_{\alpha}$$
   is a maximal nilpotent subalgebra of $L$.

3. Let $L$ be a finite dimensional Lie algebra. Show that $U(L)$ has no zero-divisors.

4. Let $L$ be a simple Lie algebra and let $\{\alpha_1, \ldots, \alpha_\ell\}$ be a base of a root system of $L$ with respect to a maximal toral subalgebra. Show that there is a root $\theta = \sum_{i=1}^{\ell} d_i \alpha_i$, such that for any other root $\beta = \sum_{i=1}^{\ell} c_i \alpha_i$ one has $c_i \leq d_i$ for $i = 1, \ldots, \ell$.

5. Let $L$ be a semi-simple Lie algebra, $H$ a maximal toral subalgebra, $\Delta = \{\alpha_1, \ldots, \alpha_\ell\}$ a base of the root system of $(L, H)$. Fix $i$, $1 \leq i \leq \ell$, and let $\alpha$ be a root of $(L, H)$, $\alpha \neq \pm \alpha_i$. Show that 
   $$\sum_k (\alpha + k\alpha_i, \alpha_i) = 0$$
   where the sum is taken over all $k$ such that $\alpha + k\alpha_i$ is a root of $(L, H)$.