1. Let the set of $n \times n$ real matrices be normed by $\| (a_{jk}) \|^2 = \sum_{j,k=1}^n a_{jk}^2$ and consider the subspaces $X$ of orthogonal matrices and $Y$ of matrices with determinant 1. Show that $X$ and $Y$ are complete and that $X$ is compact but not $Y$.

2. Let $\mathbb{R}$ and $\mathbb{R}^2$ be provided with their usual topologies and consider their subspaces $X = (0, 1)$ respectively $Y = (0, 1) \times (0, 1)$. Show that $X$ is not homeomorphic to $Y$.
   
   Hint: Compare connectivity properties of $X$ and $Y$!

3. Suppose $X$ and $Y$ are metric spaces and $f : X \to Y$ is bijective and continuous, and $f^{-1}$ is uniformly continuous. Show that if $X$ is complete, then so is $Y$.

4. Let $X$ be the algebra of continuous, complex-valued functions of $n$ variables $x_1, \ldots, x_n$ which are periodic with period $2\pi$ in each variable and provided with the maximum norm. Show that the subalgebra generated by the functions $1, e^{\pm ix_1}, \ldots, e^{\pm ix_n}$ is dense in $X$.

5. Suppose $(X, d)$ is a complete, connected metric space with the property that every continuous function $f : X \to \mathbb{R}$ is uniformly continuous. Show that $X$ is compact.
   
   Hint: Show that if $X$ is not compact, then there exists an $\varepsilon > 0$ such that there is an infinite sequence of pairwise disjoint balls of radius $\varepsilon$ in $X$. Then construct a continuous function which is not uniformly continuous using Urysohn’s lemma.

GOOD LUCK!