

# Formelblad för Funktionsteori

Matematik LTH ht 2016

## Fourierserier

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{ik\Omega t} = c_0 + \sum_{k=1}^{\infty} a_k \cos k\Omega t + b_k \sin k\Omega t \quad \Omega T = 2\pi$$

$$c_k = \frac{1}{T} \int_{\text{period}} e^{-ik\Omega t} f(t) dt \quad \begin{cases} a_k = \frac{2}{T} \int_{\text{period}} \cos(k\Omega t) f(t) dt \\ b_k = \frac{2}{T} \int_{\text{period}} \sin(k\Omega t) f(t) dt \end{cases}$$

$$\begin{cases} a_k = c_k + c_{-k} \\ b_k = i(c_k - c_{-k}) \end{cases} \quad \begin{cases} c_k = \frac{1}{2}(a_k - ib_k) \\ c_{-k} = \frac{1}{2}(a_k + ib_k) \end{cases}$$

## Parsevals formel

$$\frac{1}{T} \int_{\text{period}} \overline{f(t)} g(t) dt = \sum_{k=-\infty}^{\infty} \overline{c_k(f)} c_k(g)$$

$$\frac{1}{T} \int_{\text{period}} |f(t)|^2 dt = \sum_{k=-\infty}^{\infty} |c_k(f)|^2$$

$$\frac{1}{T} \int_{\text{period}} |f(t)|^2 dt = |c_0|^2 + \frac{1}{2} \sum_{k=1}^{\infty} (|a_k|^2 + |b_k|^2)$$

## Halvperiodutvecklingar

### Cosinusserie

$$f(x) = c_0 + \sum_{k=1}^{\infty} \alpha_k \cos\left(\frac{k\pi}{L}x\right)$$

$$\alpha_k = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{k\pi}{L}x\right) dx$$

$$c_0 = \frac{1}{L} \int_0^L f(x) dx$$

### Sinusserie

$$f(x) = \sum_{k=1}^{\infty} \beta_k \sin\left(\frac{k\pi}{L}x\right)$$

$$\beta_k = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{k\pi}{L}x\right) dx$$

## Kvot- och rotkriteriet

$$\kappa = \lim_{k \rightarrow \infty} \left| \frac{a_{k+1}}{a_k} \right| \quad \kappa < 1 \implies \sum_k a_k \text{ konvergent} \quad \kappa > 1 \implies \sum_k a_k \text{ divergent}$$

$$\rho = \lim_{k \rightarrow \infty} \sqrt[k]{|a_k|} \quad \rho < 1 \implies \sum_k a_k \text{ konvergent} \quad \rho > 1 \implies \sum_k a_k \text{ divergent}$$

## Potensserier

$$\begin{aligned}
(1+z)^\alpha &= \sum_{k=0}^{\infty} \binom{\alpha}{k} z^k & \frac{1}{1-z} = \sum_{k=0}^{\infty} z^k & e^z = \sum_{k=0}^{\infty} \frac{1}{k!} z^k \\
\cos z &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} z^{2k} & \sin z = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} z^{2k+1} & \operatorname{Log}(1+z) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} z^k \\
\arctan z &= \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} z^{2k+1} & f(z) = \sum_{k=0}^{\infty} c_k (z-a)^k, \text{ där } c_k = \frac{f^{(k)}(a)}{k!}
\end{aligned}$$

## Cauchys integralformel

$$f^{(k)}(a) = \frac{k!}{2\pi i} \int_C \frac{f(z)}{(z-a)^{k+1}} dz$$

## Residyrregler

1. Om  $f(z) = (z-a)^{-N}g(z)$  så är  $\operatorname{Res}_{z=a} f(z) = \frac{g^{(N-1)}(a)}{(N-1)!}$
2. Om  $f(z) = (z-a)^{-N}g(z)$  och  $g(z) = \sum_{k=0}^{\infty} c_k (z-a)^k$  så är  $\operatorname{Res}_{z=a} f(z) = c_{N-1}$
3.  $\operatorname{Res}_{z=a} f(z) = \lim_{z \rightarrow a} (z-a)f(z)$
4.  $\operatorname{Res}_{z=a} \frac{f_1(z)}{f_2(z)} = \frac{f_1(a)}{f'_2(a)}$

## Funktionsserier

$$\left. \begin{array}{l} |u_k(t)| \leq m_k, \quad t \in I \\ \sum_k m_k \text{ konvergent} \end{array} \right\} \implies \sum_k u_k(t) \text{ likformigt konvergent på } I$$

$$\left. \begin{array}{l} \sum_k u_k(t) \text{ likformigt konvergent} \\ u_k(t) \text{ kontinuerliga} \end{array} \right\} \implies \sum_k u_k(t) \text{ kontinuerlig}$$

$$\left. \begin{array}{l} \sum_k u_k(t) \text{ konvergent} \\ \sum_k u'_k(t) \text{ likformigt konvergent} \end{array} \right\} \implies \frac{d}{dt} \left( \sum_k u_k(t) \right) = \sum_k u'_k(t)$$

$$\left. \begin{array}{l} \sum_k u_k(t) \text{ likformigt konvergent på } I \\ u_k \text{ kontinuerliga, } I \text{ begränsad} \end{array} \right\} \implies \int_I \left( \sum_k u_k(t) \right) dt = \sum_k \left( \int_I u_k(t) dt \right)$$