LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

FORMELBLAD
SYSTEM OCH TRANSFORMER

$$\theta(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Fourierserier:

(1)
$$\begin{cases} f(t) &= \sum_{k=-\infty}^{+\infty} c_k(f) e^{ik\Omega t} \\ c_k(f) &= \frac{1}{T} \int_{\text{period}} f(t) e^{-ik\Omega t}\, dt \end{cases}, \quad \Omega T = 2\pi$$

(2)
$$c_k(f') = ik\Omega c_k(f) \quad \text{(distributionsderivata)}$$

Parsevals formel:

(3)
$$\frac{1}{T} \int_{\text{period}} \overline{f(t)} g(t)\, dt = \sum_{k=-\infty}^{+\infty} \overline{c_k(f)} c_k(g)$$

Fouriertransformer:

(4)
$$\mathcal{F} f(\omega) = \hat{f}(\omega) = F(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} f(t)\, dt$$

(5)
$$f(t) = (\mathcal{F}^{-1} F)(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega t} F(\omega)\, d\omega$$

Parsevals formel:

(6)
$$\int_{-\infty}^{+\infty} \overline{f(t)} g(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \overline{F(\omega)} G(\omega)\, d\omega$$

$$\overset{\mathcal{F}}{\longmapsto}$$

(7)	$\lambda f(t) + \mu g(t)$	$\lambda F(\omega) + \mu G(\omega)$
(8)	$f(at)$	$\frac{1}{\lvert a\rvert} F\left(\frac{\omega}{a}\right)$
(9)	$f(t - t_0)$	$e^{-it_0\omega} F(\omega)$
(10)	$e^{i\omega_0 t} f(t)$	$F(\omega - \omega_0)$
(11)	$f'(t)$	$i\omega F(\omega)$
(12)	$tf(t)$	$i\frac{d}{d\omega} F(\omega)$
(13)	$f * g(t)$	$F(\omega)G(\omega)$
(14)	$\delta(t)$	1
(15)	1	$2\pi\delta(\omega)$
(16)	$e^{-t}\theta(t)$	$\frac{1}{1 + i\omega}$
(17)	$e^{-\lvert t\rvert}$	$\frac{2}{1 + \omega^2}$
(18)	$\frac{1}{1 + t^2}$	$\pi e^{-\lvert\omega\rvert}$
(19)	e^{-t^2}	$\sqrt{\pi} e^{-\omega^2/4}$
(20)	$\theta(t + a) - \theta(t - a)$	$2\frac{\sin a\omega}{\omega}$

Laplacetransformer:

(21) $$\mathcal{L}f(s) = \mathcal{L}_{II}f(s) = \int_{-\infty}^{+\infty} e^{-st} f(t)\, dt, \quad \alpha < \operatorname{Re} s < \beta, \quad s = \sigma + i\omega$$

(22) $$\mathcal{F}f(\omega) = \mathcal{L}_{II}f(i\omega)$$

(23) $$\mathcal{L}_I f = \mathcal{L}_{II}(f\theta)$$

$$\overset{\mathcal{L}_{II}}{\longmapsto}$$

(24)	$\lambda f(t) + \mu g(t)$	$\lambda F(s) + \mu G(s)$
(25)	$f(at)$	$\frac{1}{\lvert a\rvert} F\left(\frac{s}{a}\right)$, a reellt
(26)	$f(t - t_0)$	$e^{-t_0 s} F(s)$
(27)	$e^{at} f(t)$	$F(s - a)$
(28)	$f'(t)$	$sF(s)$
(29)	$tf(t)$	$-\frac{d}{ds} F(s)$
(30)	$f * g(t)$	$F(s)G(s)$
(31)	$f'(t)\theta(t)$	$s\mathcal{L}_{II}(f\theta)(s) - f(0)$
(32)	$\delta(t)$	1
(33)	$\theta(t)$	$\frac{1}{s}, \quad \sigma > 0$
(34)	$\theta(t) - 1 = -\theta(-t)$	$\frac{1}{s}, \quad \sigma < 0$
(35)	$t^k \theta(t)$	$\frac{k!}{s^{k+1}}, \quad \sigma > 0$
(36)	$\sin(bt)\,\theta(t)$	$\frac{b}{s^2 + b^2}, \quad \sigma > 0, \quad b$ reellt
(37)	$\cos(bt)\,\theta(t)$	$\frac{s}{s^2 + b^2}, \quad \sigma > 0, \quad b$ reellt
(38)	e^{-t^2}	$\sqrt{\pi} e^{s^2/4}$

(39) $$h(t) = Ce^{tA}B\theta(t) + D\delta(t)$$

(40) $$H(s) = C(sI - A)^{-1}B + D$$

Distributioner:

(41) $$\frac{d\theta_a}{dt} = \delta_a$$

(42) $$f(t)\delta_a(t) = f(a)\delta_a(t)$$

(43) $$f(t)\delta_a'(t) = f(a)\delta_a'(t) - f'(a)\delta_a(t)$$

Invers Laplacetransformation

Om $f(t)$ har den rationella funktionen $F(s)$ som Laplacetransform så är

(44) $$f(t) = \begin{cases} \sum\limits_{\operatorname{Re} s<\sigma} \operatorname{Res}(e^{st}F(s)), & t > 0 \\ -\sum\limits_{\operatorname{Re} s>\sigma} \operatorname{Res}(e^{st}F(s)), & t < 0 \end{cases}$$

Residyregler

1. Om $f(z) = (z-a)^{-N}g(z)$ så är $\operatorname*{Res}_{z=a} f(z) = \dfrac{g^{(N-1)}(a)}{(N-1)!}$
2. Om $f(z) = (z-a)^{-N}g(z)$ och $g(z) = \sum_{k=0}^{\infty} c_k(z-a)^k$ så är $\operatorname*{Res}_{z=a} f(z) = c_{N-1}$
3. $\operatorname*{Res}_{z=a} f(z) = \lim_{z\to a}(z-a)f(z)$
4. $\operatorname*{Res}_{z=a} \dfrac{f_1(z)}{f_2(z)} = \dfrac{f_1(a)}{f_2'(a)}$