Image Analysis, Handin 1

These are distributed during the lecture September 6, 2012 and has to be finished by September 13, 2013. Written solutions are handed in either (i) at the lectures or (ii) to the box entitled 'inlämningsuppgift bildanalys' in the corridor on the third floor across from room 333 of the math building.

Note: Write your solutions neatly and explain your calculations. (For example, if you provide source code, then it needs to be described). All exercises should be done *individually.*

1. Histogram equalization

An image (in a continuous representation) has gray level histogram

$$p_r = \frac{3}{2}\sqrt{r}, \quad r \in [0,1] \ .$$

What gray level transform $s = T(r)$ should be used so that the resulting histogram p_s is uniform, i.e.

$$p_s = 1, \quad s \in [0,1] \ ?$$

2. Image bases

We would like to represent 2×2 images in a new orthonormal basis. We would like the first three basis images to be

$$\Psi_1 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \quad \Psi_2 = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \quad \text{and} \quad \Psi_3 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \ .$$

Determine a fourth basis vector Ψ_4 such that $(\Psi_1, \Psi_2, \Psi_3, \Psi_4)$ becomes an orthonormal basis.

What are the coordinates for the image

$$F = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

in the new basis?

3. Image compression

A small camera delivers low resolution images with 3×4 pixels. Before transmitting the image to a computer, one would like to compress the images consisting of 12 intensities to 4 numbers. After studying numerous images and using principal component analysis one has determined that the following four images represent typical images well,

$$\phi_1 = \frac{1}{3}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \phi_2 = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \phi_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \phi_4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{pmatrix}.$$

Show that these four images are orthonormal in the scalar product

$$(f, g) = \sum_i \sum_j f(i,j) g(i,j) \ .$$

How should we determine the four numbers (coordinates) x_1, x_2, x_3, x_4 such that the approximate image

$$f_a = x_1\phi_1 + x_2\phi_2 + x_3\phi_3 + x_4\phi_4$$

is as close to f as possible, i.e. such that $|f - f_a|^2 = (f - f_a, f - f_a)$ is as small as possible? Calculate x_1, x_2, x_3, x_4 for the image

$$f = \begin{pmatrix} -2 & 6 & 3 \\ 13 & 7 & 5 \\ 7 & 1 & 8 \\ -3 & 4 & 4 \end{pmatrix} \ .$$

4. Segmentation

On the web page for the course in image analysis there is a zip-file. By downloading and unpacking the file one obtains a folder with (i) examples of images of text and (ii) matlab-scripts.

Study the script `inl1_stomme.m`, that reads in one of the images. The image contain text (dark against a light background) Write a matlab program `im2segment` that takes such an image matrix I as input and returns a segmentation, i.e. a set of images $S = (S_1, \ldots, S_n)$ one for each letter in the image. Each such image matrix S_i should be a matrix with ones at the pixels for that letter and zeroes for all other pixels. A suggestion is to use a so called 'cell array' in matlab as the data structure for the output.

```
S = cell(1,n);
S{1} = bild1;
...
S{n} = bildn;
```

A suggested name for the script is

```
function S = im2segment(I);
```

In the written solution to the segmentation problem, supply both code (e.g. matlab code) and a printout the results of using your algorithm, i.e. supply examples of input data (e.g. as images) and result after applying your segmentation algorithm (e.g. as a number of images).