Calculus of Variations, LP3–4, Spring term 2019 (FMAN25/MATC25/FMA201F, 7,5hp)

Teacher: Niels Chr. Overgaard (NCO), tel. 046-222 85 32, epost nco@maths.lth.se, rum MH:551B.

Lectures: (NCO) Time and place, see TimeEdit (search for one of the course codes.)

Introduction meeting: (NCO) Monday 21 January 2019 at 17:15 in MH:H.

Literature:
M. Kot: *A First Course in the Calculus of Variations*,
Student Mathematical Library Vol. 72. Amer. Math. Soc. 2014.
First two chapters available from http://www.ams.org/bookstore-getitem/item=STML-72
Matematiska institutionen: *Extra study material in the form of notes and exercises.*
(If you have access to Mesterton-Gibbons book, *A Primer on the Calculus of Variations and Optimal Control*, then that will work equally well. I know both books.)

Examination: Two compulsory assignments and an oral examination ($\approx$ 30 min.)

Webpage: http://www.maths.lth.se/course/newcalcvar/2019/

Plan for the lectures: TBA

Description. The official birth date of *the calculus of variations* is set by many to be June 1696. In that month appeared in the pages of Leibniz' journal *Acta Eruditorum* a note by the eminient Swiss mathematician John (Johan) Bernoulli in which he set his collague mathematicians the challenge: To determine the curve, lying in a vertical plane, along which a bead would slide under influence of gravity, from a given initial point to a terminal point, in the shortest possible time. John himself, of course had his solution ready. Five other persons submitted solutions: Johans elder brother James (Jakob) Bernoulli, Leibniz, Tschirnhaus, l'Hopital and Newton. It is said that Newton solved the problem overnight and submitted his solution by first mail the next morning.

During the 18th century the subject was greatly developed by Euler, Lagrange and Legendre. During the 19th century the theory was extended by Jacobi and Hamilton and put on a firm footing by Weierstrass and his pupils. During the first half of the 20th century calculus was developed further by Hilbert and Charatheodory in Germany and the Chicago school in the USA, represented by Bolza, Bliss and Hestenes. After the second world war the calculus of variations transformed into the modern theory of optimal control.

The goal of the present course is to cover the *classical* theory of the calculus of variations. This covers the subject from its birth in the 17th century (even back to the antiquity, if we regard Dido's problem as an early event in the calculus of variations) up to the 20th century (about 1920 or 1930). The culmination of the course will be the proof of Weierstrass' sufficient conditions for optimality by use of Hilbert's invariant integral. At the end of the course we'll briefly touch upon the obstacle problem and other convex variational problems. Moreover, given time, the problems of Lagrange and Mayer are considered and the (general) multiplier rule for such problems is stated, leading us all the way to the threshold of optimal control theory.