Reconstruction and Optimization

- Objective Function: Reconstruction Error
- Principles of Local Optimization
- Least Squares Optimization
- Non-Linear Least Squares
Under the assumption that image points are corrupted by Gaussian noise, minimize the reprojection error.

The reprojection error

In regular coordinates \((x = (x, y))\) the projection is

\[
\begin{pmatrix}
    p_1^T x & p_2^T x \\
    p_3^T x & 0
\end{pmatrix},
\]

\(p_1, p_2, p_3\) are the rows of \(P\).

The reprojection error is

\[
\| \left( x - \frac{p_1^T x}{p_3^T x}, y - \frac{p_2^T x}{p_3^T x} \right) \|^2.
\]
Calibrated Structure and Motion

Given image projections \( \{(x_{ij}, y_{ij})\} \) \((i = \text{point nr}, j = \text{image nr})\), find 3D points \( X_i \) and cameras \( P_j = \begin{bmatrix} R_j & t_j \end{bmatrix} \) such that

\[
\sum_{ij} \left\| \left( x_{ij} - \frac{P_j^1 X_i}{P_j^3 X_i}, y_{ij} - \frac{P_j^2 X_i}{P_j^3 X_i} \right) \right\|^2,
\]

is minimized.

- Complicated non linear expression.
- No closed form solution.
Pick a starting point.
Approximate the function using 2nd order Taylor expansion and minimize.
Minimizing Reprojection Error, Locally

- Repeat.
Minimizing Reprojection Error, Locally

Newtons method.
Different starting point.
Minimizing Reprojection Error, Locally

Different starting point.
Minimizing Reprojection Error, Locally

Leads to local minimum.
Minimizing Reprojection Error, Locally

Third starting point, leads to local maximum.
Minimizing Reprojection Error, Locally

Why not just sample the function?
One dimensional function:
Why not just sample the function?
One dimensional function:

Sample 10 points, pick lowest value. Probably works.
Minimizing Reprojection Error, Locally

Why not just sample the function?
Two dimensional function:

\[ 10^2 \text{ samples}. \]
Minimizing Reprojection Error, Locally

Why not just sample the function?
Three dimensional function:

$10^3$ samples.
How many variables do we have?

The cathedral dataset:
- 480 camera matrices $[R_i \ t_i]$. Rotation part 3 dof, translation part 3 dof. Totally: $480(3 + 3) = 2880$.
- 91178 3D points. 3 dof each. Totally: $91178 \cdot 3 = 273534$
Local Optimization

See lecture notes.
Steepest Descent

See lecture notes.
Demonstration...
Gauss-Newton

See lecture notes. Demonstration...
Levenberg-Marquard

See lecture notes.
Demonstration...