Iterative Solution of Large Scale Systems in Scientific Computing

Assignment 5

Problem 1
Consider the following variant of the multigrid method: On the coarsest grid, the smoother is applied instead of the direct solver. A V-cycle is used and only presmoothing is employed. Now let the smoother be given by \( S_l(x_l, b_l) = S_l^x x_l + S_l^b b_l \). If the smoother is only applied once, show that for a three-level scheme the iteration matrix of \( MG(x_2, b_2, 2) \) is given by

\[
M = S_2^x - P_{2,1} \left( S_1^b + P_{1,0} S_0^b R_{0,1} (I - A_1 S_1^b) \right) R_{1,2} A_2 S_2^x.
\]

Problem 2
Let \( G_l \) denote the iteration matrix of the smoother \( S_l(x_l, b_l) \). Show that the iteration matrix \( M_l(\nu_1, \nu_2) \) of the multigrid method \( MG(x_l, b_l, l) \) using \( \nu_1 \) presmothings and \( \nu_2 \) postsmoothings can be defined recursively as

\[
M_0(\nu_1, \nu_2) = 0,
M_k(\nu_1, \nu_2) = G_k^{\nu_2} (I - P_{k, k-1} (I - M_{k-1}(\nu_1, \nu_2)) A_{k-1}^{-1} R_{k-1,k} A_k) G_k^{\nu_1}, \quad k = 1, \ldots, l.
\]

Problem 3
Consider the problem

\[-u''(x) = 4\pi^2 \sin \pi x^2, \quad x \in [0,1],
\]

\[u(x) = 0, \quad x = 0, x = 1.\]

Implement a 2-grid method with Jacobi relaxation as smoother for this problem. Vary \( \Delta x \) and the number of pre- and postsMOOTHING steps.

Return: Wednesday, May 9th, in class