No aids allowed. Use the distributed paper sheets and write only on one side. Fill in the cover sheet completely and write your initials on each sheet. Write legibly (in Swedish or English). Motivate your conclusions clearly and concisely; draw a picture if appropriate.

1. Find all fixed points of the system
\[
\begin{align*}
x' &= x^2 + xy \\
y' &= x^2 + 2xy + x + 2y + 2
\end{align*}
\]
and determine their stability properties.

2. Prove that the boundary value problem
\[
\begin{align*}
y''(x) + 4y(x) &= f(x), \quad 0 < x < 1, \\
y(0) &= y(1) = 0
\end{align*}
\]
has a unique solution for any \( f \in C[0, 1] \). Determine Green’s function and express the solution in the form of an integral.

3. Prove that the origin is an asymptotically stable fixed point for the autonomous system
\[
\begin{align*}
x' &= 2y^3 \\
y' &= -x^2y - x.
\end{align*}
\]
Show that any solution converges to the origin as \( t \to \infty \).

4. Show that the equation \( x'' - x'(1 - 7(x')^2 - 5x^2) + x = 0 \) has a non-constant periodic solution.

5. Assume that \( q \) is a continuous function with \(-4 \leq q(x) < -1\), when \( 0 \leq x \leq \pi \). Show that the eigenvalue problem
\[
\begin{align*}
-u'' + q(x)u &= \lambda u, \quad 0 \leq x \leq \pi, \\
u(0) &= u(\pi) = 0
\end{align*}
\]
has precisely one (strictly) negative eigenvalue. Is the conclusion still valid if we only know that \(-4 \leq q(x) \leq -1\)?