



LUND
UNIVERSITY

Written Examination
Flervariabelanalys 1 MATB21
August 16, 2016
Time: 8.00–13.00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Use the papers provided by the department. Write clearly with short and concise motivations. Illustrate with a figure when necessary.

1. Find the largest and smallest values of $f(x, y) = x^2 + 2y^2 - x$ over the unit disk $x^2 + y^2 \leq 1$.
2. Prove that the equation $e^{z-1} + 2zy + x - 3y^3 = 0$ defines z as a C^1 -smooth function of x and y in a neighbourhood of the point $P = (0, 1, 1)$. Determine the partial derivatives $z'_x(0, 1)$ and $z'_y(0, 1)$.
3. Calculate the following integrals:
 - a) $I_1 = \iint_{D_1} \frac{2x}{x^2+y} dx dy$ where D_1 is the domain bounded by the curves $x = 0$, $x = 1$, $y = 0$, $y = 4 - x^2$.
 - b) $I_2 = \iiint_{D_2} z^3 dV$, where $D_2 : x, y \geq 0$, $x^2 + y^2 \leq 1$, $0 \leq z \leq \sqrt{y}$.

4. Define, for $x > 0$,

$$F(x) = \int_0^x e^{y-x} \cos(y^2) dy.$$

Prove that $u = F$ solves the initial value problem

$$u'(x) + u(x) = \cos(x^2), \quad u(0) = 0.$$

5. Transform the differential equation

$$yu'_x(x, y) - xu'_y(x, y) = xyu(x, y)$$

by introducing the new variables

$$s = x^2 + y^2, \quad t = e^{-x^2/2}.$$

Then determine the general solution to the differential equation.

6. Compute (as a function of r) the maximum of the function

$$f(x, y) = (3x + 4y)e^{-(x^2+y^2)/2}$$

over the closed disk $x^2 + y^2 \leq r^2$.