



LUND
UNIVERSITY

Written Examination
Analysis of Several Variables 1
Saturday 29 October 2016
Duration: 14.00–19.00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

No aids are allowed. Please write on one page only, give concise and short arguments and draw figures when applicable. The result will be posted at the latest at 12.00 on Wednesday, November 2.

1. Determine all points where the function

$$f(x, y) = x^2 + 4xy + 24y^2 - y^5$$

has a local maximum or minimum.

2. Calculate the integral

$$\iint_D \frac{x}{1 + x^2 + y^2} dx dy,$$

where D is the area defined by the inequalities $x^2 + y^2 \leq 1$, $x \geq \sqrt{3}|y|$.

3. Find the maximum and the minimum of the function

$$f(x, y) = \frac{4x - 3}{1 + x^2 + y^2}$$

in the disk

$$x^2 + y^2 \leq 9.$$

4. Show that the equation

$$x^3 y + x \ln y + y^3 = 9$$

defines y as a function of x in a neighborhood of $(2, 1)$ and calculate $y'(2)$ and $y''(2)$.

5. Determine a continuous function f such that

$$\int_0^x ((x-t)^2 - 2)f(t) dt = 4x.$$

6. Compute the triple integral

$$\iiint_S |x^2 + y^2 - z^2| dx dy dz,$$

where S is the unit sphere, $x^2 + y^2 + z^2 \leq 1$