LUND UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Written Examination
Flervariabelanalys 1 MATB21
March 18, 2017
Time: 14.00–19.00

Use the papers provided by the department. Write clearly with short and concise motivations. Illustrate with a figure when necessary.

1. Consider the function $f(x,y) = xy(3-x-y)$. Find the largest value of $f(x,y)$ over the first quadrant $x, y \geq 0$.

2. Evaluate the generalized integral

$$\iiint_D \frac{x^2 z}{x^2+y^2+z^2} e^{-x^2-y^2-z^2}\, dxdydz,$$

where $D:\ z \geq \sqrt{3(x^2+y^2)}$.

3. Consider the system of equations

$$\begin{cases} x^2+y^2-z^2 &= 2 \\ x+y-2e^z &= 0 \end{cases}.$$

Verify that $(x,y,z) = (1,1,0)$ is a solution. Show that the system can be solved for y and z as smooth functions of x near the point $(1,1,0)$ and compute $y'(1)$ and $z'(1)$.

4. Find the general (C^2-smooth) solution $u(x,y)$ to the problem

$$\cos^2(x) u''_{xy} + u''_{yy} = 0.$$

It is helpful to introduce the new variables $s = \tan x$, $t = y - \tan x$. Also find a particular solution such that $u(x,0) = \tan^2 x$ and $u'_y(x,0) = 0$.

5. Find an equation of the curve in the xy-plane which passes through $(1,1)$ and which intersects all level curves of the function $f(x,y) = x^2 e^y$ orthogonally.

6. Let $u(x,y)$ be a C^∞-smooth function. Given $\epsilon > 0$ let $D_\epsilon:\ x^2+y^2 < \epsilon^2$ be the ϵ-neighbourhood of the origin. Denote the average value of u over D_ϵ by

$$A(\epsilon, u) = \frac{1}{\pi\epsilon^2} \iint_{D_\epsilon} u\, dA.$$

a) Define the second-order Taylor polynomial $P_2(x,y)$ of u about $(0,0)$. What does Taylor's formula say about the size of the difference $u(x,y) - P_2(x,y)$ as $(x,y) \to (0,0)$?
b) Assume that $\Delta u(0,0) > 0$ where $\Delta u = u''_{xx} + u''_{yy}$ is the Laplacian. Prove that the inequality $A(\epsilon, u) > u(0,0)$ holds for all sufficiently small $\epsilon > 0$.

Comment. A function which satisfies $\Delta u > 0$ at a point is said to be *subharmonic* there.