



LUND
UNIVERSITY

Written Examination
Flervariabelanalys 1 MATB21
August 16, 2017
Time: 8.00–13.00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Use the papers provided by the department. Write clearly with short and concise motivations. Illustrate with a figure when necessary.

1. Determine local extreme values for the function $f(x, y) = 8xy - 4x^2y - 2xy^2 + x^2y^2$.
2. Find the integral

$$\iiint_R (x^2 + y^2 + z^2) dV$$

where R is the region that lies above the cone $z = c\sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = a^2$. (Here a and c are positive constants.) Simplify as far as possible.

3. Compute the length of the plane curve $\gamma : (x, y) = (t^2, t^3)$, $-2 \leq t \leq 1$.
4. Let $u(x, y)$ be a C^1 -smooth function. Transform the equation

$$(x + y) \frac{\partial u}{\partial x} - (x - y) \frac{\partial u}{\partial y} = 0$$

by introducing the new variables $s = \ln \sqrt{x^2 + y^2}$, $t = \arctan \frac{y}{x}$. Use this to write down the general solution to the equation.

5. Let f be a differentiable function of one variable. Prove that all tangent planes to the surface $z = xf(y/x)$, $x \neq 0$, pass through the origin.
6. Show that the equation $x + 2y + z + e^{2z} = 1$ has a smooth solution $z = z(x, y)$ defined in a neighbourhood of the origin $x = y = z = 0$. Find the Taylor polynomial of degree 2 of the function $z(x, y)$ about the point $x = y = 0$.