LUND UNIVERSITY

**Written Examination**
**Discrete mathematics, 7.5 credits**
**Friday January 14, 2011**
**Time: 08.00-13.00**

Centre for Mathematical Sciences
*Mathematics, Faculty of Science*

*Use only the distributed paper sheets; write only on one side, and no more than one problem per sheet. Fill in the cover form fully and initialize each sheet. Write legibly. Give clear and brief arguments. Draw a picture if this helps.*

*For those who pass the written exam, an oral exam will take place January 18. The exact time and place for each student will be posted on a list at the department 08:00 January 17. Please indicate on the answer sheet your preferred time.*

**1.** Solve the recurrence relation

$$a_{n+2} - 4a_{n+1} - 5a_n = 30 \cdot 5^n,$$

for $n \geq 0$, when $a_0 = 1$ and $a_1 = 10$.

**2.** Determine the number of integer solutions to the equation

$$x_1 + x_2 + x_3 + x_4 = 20,$$

under the condition $1 \leq x_i \leq 8$ for $1 \leq i \leq 4$.

**3.** Solve the system of equations

$$\begin{aligned} 8x + 9y + 3z &\equiv 1 \pmod{11} \\ 3x + 2y + 3z &\equiv 1 \pmod{11} \\ 4x + 9y + 5z &\equiv 1 \pmod{11}. \end{aligned}$$

**4.** **a)** Determine all monic prime (irreducible) polynomials of degree 2 in $\mathbb{Z}_3[x]$.
**b)** Find a zero divisor of $x^2 + 2$ in $\mathbb{Z}_3[x]/(x^4 + 2x^3 + x + 2)$.

**5.** Show that

$$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$$

whenever $p, q$ are two different prime numbers.

**6.** For $n \leq m$, show that

$$n^m = \sum_{k=0}^{n} k! S(m,k) \binom{n}{k},$$

where $S(m,k)$ are the Stirling numbers of the second kind.