LUND UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Written Examination
Discrete Mathematics
Monday, December 19, 2011

No books, notes, computational devices, etc. are allowed. Use only paper supplied by the department. Use clear handwriting and give clear careful motivations. Fill in the form completely and write your name on each sheet of paper.

1. Find the number of positive integers n where $1 \leq n \leq 1000$ and n is *not* a perfect square, cube or fifth power.

2. Calculate the quantity of integers between 500 and 999 such that the sum of their digits is 13.

3. Let (a_n) be a sequence of real numbers such that it starts with the terms $a_0 = 0, a_1 = 1$ and each following term is the average of the two previous ones and then shifted by 1 to the right on the real line. Determine and solve the recurence relation that defines the sequence.

4. Determine the number of ways to distribute 6 balls into 5 containers if:

a) the balls are all different and the containers are all different,
b) the balls are all identical and the containers are all different,
c) the balls are all different and the containers are all identical,
d) the balls are all identical and the containers are all identical.

5. Find a simultaneous solution for the system of three congruences:

$$\begin{aligned} 2x + 1 &\equiv 3 \pmod{5} \\ x - 1 &\equiv 4 \pmod{6} \\ 3x + 2 &\equiv 1 \pmod{7}. \end{aligned}$$

6. Consider the binary linear $(6, 3)$ code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

What can be said about the error-correction capability of the encoding function defined by G? Decode the following word 110010. Can the received word 110001 be properly decoded?