

LUNDS TEKNISKA HÖGSKOLA TENTAMENSSKRIVNING
MATEMATIK

DISKRET MATEMATIK FMAA25/MATB13
2017–08–14 kl 14–19

No books, notes, computational devices, etc. are allowed. Use only paper supplied by the department. Use clear handwriting and give clear careful motivations. All answers should be fully simplified. In particular they should not contain binomial coefficients, Stirling numbers or factorials. Fill in the form completely and write your personal identifier on each sheet of paper.

1. Compute the number of integers between 1 and 100 that are divisible by at least one of the numbers 3, 7, 10.
2. Consider the ring $R = \mathbb{Z}_5[x]/(x^3 + 2x^2 + 2x + 2)$.
 - a) Compute the product $[x^2] \cdot [x^3]$ in R . (The answer should be on the form $[ax^2 + bx + c]$.)
 - b) Find out if $[x^2 + 2x]$ is invertible in R and if so determine its inverse.
 - c) Is R a field?
3. Find all integer solutions to the following systems of equations:
 - a)
$$\begin{cases} x \equiv 2 \pmod{3} \\ x \equiv 1 \pmod{5} \\ x \equiv 0 \pmod{11}. \end{cases}$$
 - b)
$$\begin{cases} x \equiv 2 \pmod{6} \\ x \equiv 1 \pmod{10} \\ x \equiv 0 \pmod{11}. \end{cases}$$
4. Use the letters in DISCRETE MATHEMATICS. (In this problem you may use factorials in your answers.)
 - a) How many strings of length 19 can you form?
 - b) How many strings of length 7 using no letter more than once?
 - c) How many strings of length 5 using no letter twice. (But the same letter may be used three times.)

5. Consider the linear code C over \mathbb{Z}_2 generated by

$$G = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

- a) Determine the separation of C .
 - b) Which of the words $w_1 = (1111)$, $w_2 = (0111)$ and $w_3 = (0101)$ are in C ?
 - c) Construct a $[8,4]$ code over \mathbb{Z}_2 with separation 4.
6. Let a_n be the number of words of length n in the letters A, B, C not containing the substring CC .
- a) Compute a_1, a_2, a_3 and a_4 .
 - b) Find a general expression for a_n .

Good Luck!