No books, notes, computational devices, etc. are allowed. Use clear handwriting and give clear careful motivations. All answers should be fully simplified, but may contain factorials or powers. In particular they should not contain binomial coefficients or Stirling numbers. Write your date of birth on each sheet of paper.

1. In how many ways can 12 identical chocolate bars be distributed between three teenagers: John, Lisa and Dana if
   a) There are no restrictions on the number of bars each person gets?
   b) If John should get an odd number of bars and Lisa and Dana should get at least two bars each?
   c) If John should get an odd number of bars, Lisa an even number of bars and Dana should get at least four bars?

2. In how many ways can the digits 1, 2, 3, 4, 5, 6 be permuted if
   a) no digit should end up in the correct position? (More precisely: digit $k$ is not in position $k$ of the string for $k = 1, 2, 3, 4, 5, 6$)
   b) exactly one digit should end up in the correct position?
   c) Which one of the types of sequences described in a and b is most likely if you generate a random permutation? (Assuming all permutations have the same probability of coming up.)
   d) If we look at the digits 1, 2, 3, 4, 5, 6, 7 instead of 1, 2, 3, 4, 5, 6 what is the answer to question c?

3. a) Find the orders of all non-zero elements in $\mathbb{Z}_7$. How many primitive elements are there?
   b) Find a $p$ and an $h(x)$ that makes $\mathbb{Z}_p[x]/(h(x))$ a field with 25 elements.
   c) Find the multiplicative inverses of five different non-zero elements of the field you found in b.

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4. Consider the linear code $C$ over $\mathbb{Z}_3$ with control matrix

$$H = \begin{pmatrix}
1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 2 \\
0 & 1 & 0 & 1 & 2 & 0 & 0 & 1 & 1 & 1 & 2 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1
\end{pmatrix}$$

a) What is the dimension of $C$?

b) Find the separation of $C$.

c) For each of the words $w_1 = (1111111111111)$ and $w_2 = (1212121212121)$ check if it is a code word. If not correct the word if possible. (Remember to explain why it can or cannot be corrected.)

d) Is $C$ a perfect code?

5. Find a second degree recurrence relation with polynomial right hand side having $a_n = 4 \cdot 2^n + 7 \cdot (-1)^n + n^2 - 7$ as one of its solutions. Also state initial conditions that makes $a_n$ the only solution of your recurrence.

6. Christine is going to her summer cabin to spend the whole month of July there. She brings a big basket with 50 apples. Each day in July she will eat at least one apple. (She does not have to eat all apples in the basket before going back home.)

a) Show that there is a span of consecutive days such that she eats exactly 11 apples in total during that time.

b) If you replace July by February, is the statement still true? (Provide a proof or a counterexample.)

c) If you replace July by June, is the statement still true? (Provide a proof or a counterexample.)

*Note: February has 28 days, June 30 days and July 31 days.*