LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
DISKRET MATEMATIK
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LÖSNINGAR

1. We can use inclusion/exclusion here. Let S all arrangements of the letters in COEFFICIENT and use the conditions $c_1=$ contains TEN, $c_2=$ contains FIT and $c_3=$ contains CONE. Then $S_0 - S_1 + S_2 - S_3$ arrangements contain none of the substrings. $S_0 = \frac{11!}{2!^4}$, $S_1 = N(c_1) + N(c_2) + N(c_3) = \frac{9!}{2!^3} + \frac{9!}{2!^2} + \frac{8!}{2!^2}$, $S_2 = N(c_1c_2) + N(c_1c_3) + N(c_2c_3) = \frac{7!}{2!} + 0 + 6!$ and $S_3 = N(c_1c_2c_3) = 0$ (note that TEN and CONE cannot both be substrings at the same time) we find that $S_1 - S_2 + S_3 = \frac{11!}{2!^4} - \frac{9!}{2!^3} - \frac{9!}{2!^2} - \frac{8!}{2!^2} + \frac{7!}{2!} + 0 + 6! - 0 (= \frac{6!\cdot 6533}{2} = 2351880)$

2. Multiplying the first equation by $3^{-1} = -2$ and the second by $(-1)^{-1} = -1$ we get the system

$$\begin{cases} x \equiv 3 \pmod 7 \\ x \equiv 1 \pmod{11} \end{cases}$$

This has a unique solution modulo 77 by the Chinese remainder theorem. We can find one solution $x = 45$ using standard methods with the Euclidean algorithm. All positive solutions are then given by $x = 45 + 77n$ where n is any non-negative integer.

3. This can be described as the coefficient of x^{10} in the generating function

$$g(x) = (x^4 + x^5 + x^6 + x^7 + x^8 + x^9)(1 + x + x^2 + \cdots)^3 =$$
$$= \frac{x^4(1 - x^6)}{(1 - x)^4} = (x^4 - x^{10}) \sum_{j=0}^{\infty} \binom{-4}{j} (-x)^j$$

Here the first equality comes from expressions for geomtric sums and geometric series. Adding contribution from $j = 6$ and $j = 0$ we get the coefficient $\binom{-4}{6} - 1 = 83$

4. All towers of height a_n has either a red or a blue block in the top position. There are a_{n-1} towers with red top as each comes from adding a red block to an tower of height $n-1$. Likewise we find that there are a_{n-2} towers with a blue top. Hence $a_n = a_{n-1} + a_{n-2}$. As initial conditions we may use $a_0 = 1$ and $a_1 = 1$. From the characteristic equation we find that $a_n = C(\frac{1+\sqrt{5}}{2})^n + D(\frac{1-\sqrt{5}}{2})^n$. From the initial conditions we can solve for C and D obtaining the solutions $a_n = \frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^{n+1} - \frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^{n+1}$

5. **a)** C contains 7^4 words.

b) Computing the syndromes $w_i H^t$ we get $(0,0,0), (2,2,2), (0,0,0)$ and $(2,3,2)$ for $i = 1,2,3,4$. This shows that w_2 and w_4 are code words, while w_1 and w_3 are not. The syndrome of w_2 is $2H_2$ so w_2 has only one error and can be corrected to $w_2 - (0\ 2\ 0\ 0\ 0\ 0\ 0) = (1\ 1\ 1\ 1\ 1\ 1\ 1)$. The syndrome of w_4 is not a multiple of a column of H which shows that it must contain more than one error.

c) Any 3x3 submatrix of H is a Vandermonde matrix, and then we know it has non-zero determinant implying that its columns are linearly independent. Any four columns of H are dependent (as they are in a 3 dimensional space) and therefore $d(C) = 4$ by theorem 3.10 in Andersson.

6. **a)** Count first the monic irreducible polynomials and then multiply by four as each of them has four multiples of degree two. $p(x) = x^2 + ax + b$ is irreducible if $p(0) = b, p(1) = 1+a+b, p(2) = -1+2a+b, p(-1) = 1-a+b$ and $p(-2) = -1 - 2a + b$ are all non-zero. (A degree two polynomial is irreducible if and only if it has no first degree factors which by the factor theorem is equivalent to having no zeroes.) Running through the possible values we find ten choices of (a, b) that satisfy the conditions. Hence the number of irreducible polynomials of degree two is 40.

c) The factorisation $x^4+1 = (x^2+2)(x^2-2)$ shows that x^4+1 is not irreducible. (One way to find a factorization is to use the irreducibles of degree two obtained by the method i part a.)