LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
Linjär och kombinatorisk optimering
2003-3-13 - 2000-3-21, HEMTENTA

You may use any books and computer programs (e.g. Matlab and Maple), but it is not permitted to get help from other persons. Programs and long calculations can be submitted by e-mail (ufn@maths.lth.se). The data for problems can be received from http://www.maths.lth.se/matematiklth/personal/ufn/kombopt/tentadata.html

Hand in solutions to 6 of 9 problems below. For a passing grade (3), at least 3 problems have to be solved correctly. Credits can be given for partially solved problems. Write your solutions neatly and explain your calculations. Both the content and the format of your solutions, and also how difficult problems you choose, will affect your grade.

The exam has to be handed in to the Student's Expedition in the Mathematics Department at the latest March, 21 at 5.00 p.m. Write your name, section-year (or subject for Ph.D-students), id-number, and phone number or email address on the first page, and write your name on each of the following pages. The result will be announced on the board on the first floor in the Mathematics building on March 31, and those who have given an email address will be notified that way. The oral part of the exam should take place in March-April (depending on your schedule).

Problems can have some parameters $(a, b, c, d \ldots)$. The value of the parameters should be chosen according your personal number (or simply birthday if you have not personal number) abcdeg-****. For example if your personal number is 650327-2384 then $a = 6, b = 5, c = 0, g = 7$ in your problems.

1. Solve the following linear programming problem

$$[-2\ a\ b\ d\ ]\ x \to max$$

with

$$\begin{bmatrix} 4 & -2 & 4 & 5 \\ 1 & 2 & -5 & 6 \\ 2 & -15 & 4 & 5 \\ 7 & 2 & -4 & 0 \\ 2 & 0 & -6 & 1 \\ 0 & 3 & -3 & 1 \end{bmatrix} x \leq \begin{bmatrix} 12 \\ 15 \\ -2 \\ 28 \\ -4 \\ 10 \end{bmatrix}, x \geq 0$$

for example using the two phase method. Are there any feasible solutions? If there is an optimal solution find it. Do the same for the dual problem.

2. A company has contracted for five jobs. These jobs can be performed in six of its manufacturing plants. Because of the size of the jobs, it is not feasible to assign more then one job to a particular manufacturing facility. Also, the second job J2 cannot be assigned to the third manufacturing plant P3. The cost estimates, in thousands of

dollars, of performing the jobs in the different manufacturing plants, are summarized:

	$P1$	$P2$	$P3$	$P4$	$P5$	$P6$
$J1$	50	55	42	57	48	52
$J2$	66	70	$--$	68	75	63
$J3$	$10 * a + d$	78	72	80	85	78
$J4$	40	42	38	45	46	42
$J5$	62	55	58	60	56	65

a) Formulate the problem of assigning the jobs to the plants so that the total cost is minimized.

b) Solve the problem by the transportation algorithm.

c) Solve the problem by the Hungarian assignment algorithm.

3. A new casino "WIN HERE" suggests a new game where the probability p to win is larger then half. The restrictions are as follows:

- You can play maximum 3 times.
- Every time you are playing you can use part of your money an they either will be doubled with a probability $p > \frac{1}{2}$ or lost with the probability $1 - p$.
- You cannot borrow money from others.
- You cannot leave the casino with more then 100.000 in the pocket.

Suppose that you have 30.000 with you. What is your strategy to maximize the expected result? What price should take the casino for this game to have the profit?

Hint: study first the cases of one and two games.

Example. If you use all the sum in the first game you can double it with a probability p. In this case you cannot use more then 40.000 for the second time. But if you lose the first game you be unable to play more. But maybe it was better to use only part of your money in the first game?

4. Use the critical path method to plan your future PH.D defense. Suppose that you already have obtained all necessary scientific results and you scientific adviser have decided that it is time for the defense and have chosen an opponent. Select yourself necessary events (e.g. writing, printing, sending the manuscript, the party after defense, flowers, projector, reservation the hotel for the opponent etc) and reasonable parameters (which events can be done simultaneously and for which it is necessary to wait another event. For example you should first write the manuscript, get it printed and only after that send it). Remember that some events (e.g. reading your manuscript) take a long of time. Some useful information can be found in the "Studiehandbook". Find early event times, late event times and critical events.

5. Is it possible to organize 9 tennis players of different rating is three teams A,B and C such that

a) Every team consists on 3 persons;

b) A wins against B, B against C and C against A?

It is known that in a team play everybody plays 3 single games against all three players from another team and the stronger player is always winning against the weaker. For example if $A = \{4, 7, 8\}, B = \{2, 3, 9\}$ and the larger number corresponds to the larger rating, then the result A against B will be 6:3 because 2 and 3 lose all the games, but 9 wins.

6. You need to find the most safe route to send your message from the station A to the station H. You can use the intermediate stations B-G. The probability to lose the information sending it between different stations directly is collected in the following table:

from	*to A*	*to B*	*to C*	*to D*	*to E*	*to F*	*to G*	*to H*
A	0	0.05	0.15	0.3	0.35	0.45	0.56	0.64
B	0.06	0	0.1	0.18	0.29	0.38	0.46	0.54
C	0.16	0.08	0	0.11	0.19	0.28	0.33	0.44
D	0.26	0.18	0.08	0	0.09	0.2	0.23	0.40
E	0.36	0.25	0.2	0.07	0	0.08	0.13	0.24
F	0.46	0.38	0.3	0.21	0.09	0	0.07	0.14
G	0.56	0.45	0.4	0.31	0.19	0.08	0	0.09
H	0.66	0.55	0.4	0.41	0.29	0.18	0.08	0

7. 15 peoples are members of 15 different commissions according to the following table (1 in the row i column j means that the person number i is the member of the commission number j).

	$C1$	$C2$	$C3$	$C4$	$C5$	$C6$	$C7$	$C8$	$C9$	$C10$	$C11$	$C12$	$C13$	$C14$	$C15$
$N1$	1	0	0	1	0	0	0	1	1	0	0	0	0	0	1
$N2$	c	0	1	0	1	0	0	0	1	0	0	0	1	0	0
$N3$	$1-c$	0	1	0	1	1	0	0	0	0	1	0	0	0	0
$N4$	0	0	0	1	1	0	1	1	0	0	0	0	1	0	0
$N5$	0	1	0	0	1	0	0	1	0	0	0	0	0	1	0
$N6$	0	0	1	0	0	1	0	0	1	0	1	0	0	0	0
$N7$	1	0	0	0	0	0	1	0	1	1	0	1	0	0	0
$N8$	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1
$N9$	1	1	0	0	0	1	0	0	1	1	0	1	0	0	0
$N10$	1	0	0	1	0	1	0	1	0	0	0	0	1	0	0
$N11$	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0
$N12$	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0
$N13$	0	0	0	1	1	1	0	0	0	0	0	1	1	0	1
$N14$	1	1	0	0	0	0	0	0	0	1	0	0	1	1	1
$N15$	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0

Is it possible to choose a chairman in every commission such that no one person would be the chairman in two commissions? Suggest a general algorithm to solve this problem.

8. The angry dictator Didi decided to make the test in his Consul of Wizards. The test proceeds as follows. Didi places all n wizards in a row and put a white or black hat on the head of every of them. Every wizard can see the colors of the hats on those who are standing before him, but cannot see neither his own color nor hats of those who are behind him. Every minute one (and only one) of wizards should cry one of those two colors, trying to guess the color on his head. Every wizard can cry only once. When the test is finished all wizards that guessed wrong be killed. If somebody violate the rules (for example two wizards cry in the same minute or a wizard says something else than its color or in other way try to send information to others) everybody will be killed (it was very angry dictator!).

Before the test the wizards have time to discuss their strategy. It is clear that the wizard that will be unlucky to be the last one in the row cannot be 100 percent saved, because nobody can see his color. On the other hand at least half of them may be saved for even n. The strategy is: every odd minute i the wizard number i cries the color of the wizard before him and next minute the wizard number $i+1$ repeats this color. Try to find another strategy to maximize the number of wizards that will be saved with the 100 percent guaranty.

9. The word *aabba* in the alphabet, consisting of two letters a, b is the shortest word containing all possible subwords in this alphabet of the length 2. Create the shortest word that contains all subwords of length 7 in this alphabet and the shortest word that contains all subwords in the alphabet a, b, c of length 5.