LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
Linjär och kombinatorisk optimering
2004-3-12 - 2004-3-22, HEMTENTA

You may use any books and computer programs (e.g. Matlab and Maple), but it is not permitted to get help from other persons. Programs and long calculations can be submitted by e-mail (ufn@maths.lth.se). The data for problems can be received from http://www.maths.lth.se/matematiklth/personal/ufn/kombopt/tentadata.html

Hand in solutions to 6 of 9 problems below. For a passing grade (3), at least 3 problems have to be solved correctly. Credits can be given for partially solved problems. Write your solutions neatly and explain your calculations. Both the content and the format of your solutions, and also how difficult problems you choose, will affect your grade.

The exam has to be handed in to the Student's Expedition in the Mathematics Department at the latest March, 22 at 5.00 p.m. Write your name, section-year (or subject for Ph.D-students), id-number, and phone number or email address on the first page, and write your name on each of the following pages. The result will be announced on the board on the first floor in the Mathematics building on March 31, and those who have given an email address will be notified that way. The oral part of the exam should take place in March-April (depending on your schedule).

Problems can have some parameters $(a, b, c, d \ldots)$. The value of the parameters should be chosen according your personal number (or simply birthday if you have not personal number) abcdeg-****. For example if your personal number is 650327-2384 then $a = 6, b = 5, c = 0, g = 7$ in your problems.

1. Solve the following linear programming problem

$$[-1 \; a \; b \; d \,]\; x \to max$$

with

$$\begin{bmatrix} 4 & -2 & 1 & 5 \\ 1 & 4 & -5 & 6 \\ 2 & -25 & 4 & 5 \\ 7 & 1 & -4 & 0 \\ 2 & 0 & -6 & 1 \\ 0 & 3 & -3 & 1 \end{bmatrix} x \leq \begin{bmatrix} 12 \\ 25 \\ -10 \\ 28 \\ -4 \\ 10 \end{bmatrix}, x \geq 0$$

for example using the two phase method. Are there any feasible solutions? If there is an optimal solution find it. Do the same for the dual problem.

2. An automobile manufacturer has assembly plants located in the Northwest, Midwest, and Southeast. The cars are assembled and send to major markets in the Southwest, West, East and Northeast. The appropriate distance matrix, availabilities, and de-

mands are given by the following chart.

	Sowthwest	*East*	*West*	*Northeast*	*supply*
Northwest	1000	8000	1800	2000	2000000
Midwest	400	700	900	1400	1300000
Southeast	$100a$	1200	$100(b+c)$	1100	1600000
demand	1000000	1500000	1200000	700000	

a) Assuming the cost is proportional to distance, find the the optimal shipment pattern.

b) Assuming the cost is proportional to square of distance, find the the optimal shipment pattern.

3. You want to organize a conference in combinatorial optimization. Use the critical path method to plan all the events. Suppose that you already have obtained the grant from EU and need not care too much about the money. But you need to think about the invited speakers (you should invite them and they need to answer, and if they cannot participate you maybe need to find other speakers). Select yourself other necessary events (e.g. preparing the abstracts (proceedings) of the conference, information letters, visa problems, the party after the conference, projector, coffee breaks, hotel and conference room reservation, etc) and reasonable parameters (which events can be done simultaneously and for which it is necessary to wait another event. For example you should first get the list of the invited speakers before sending the second announcement with the schedule of the talks). Remember that some events (e.g. preparing the abstracts) take a long of time.

Find early event times, late event times and critical events.

4. Consider the following sequence $\{x_k\}$ of 100 numbers.

[23, 79, 22, 98, 32, 37, 91, 99, 29, 14, 26, 42, 89, 80, 31, 12, 6, 95, 4, 61,
67, 53, 59, 25, 10, 43, 96, 97, 28, 35, 81, 33, 38, 52, 92, 56, 21, 87, 94,75,
9, 78, 55, 39, 17, 16, 90, 2, 70, 13, 20, 66, 68, 88, 57, 63, 71,48, 84, 100,
8, 15, 27, 64, 4, 24, 34, 60, 69, 83, 93, 74, 65, 76, 44, 50, 58, 86, 46, 30,
47, 82, 85, 72, 18, 62, 5, 77, 41, 1, 3, 36, 54, 11, 19, 45, 7, 49, 51 ,73]

Find a subsequence x_{k_i} of maximum possible length m with the property

$$(x_{k_i} - x_{k_{i-1}})(x_{k_i} - x_{k_{i+1}}) < 0, \ i = 2, 3 \ldots m-1.$$

(Here $k_1 < k_2 < k_3 < \ldots k_{m-1} < k_m$). Motivate you choice.

5. A read-only array (ROM) contains n integers. Find a linear-time algorithm that determines whether the array has a "majority element", and if so returns that value. An integer x is a majority element in the array if it is in k locations, where $k > \frac{n}{2}$. (If $N = \sum_{i=1}^{n} \log a_i$ the algorithm should be linear in N. The number n is large and you cannot use another array).

6. n cars and m buses are parked in the line, such that all cars stay before all buses. One place is free. You need to place buses before cars using the free place such that order between cars themselves and order between buses themselves does not changes.

Moving them cyclically you can do it in about $n(n+m)$ moves, which is quadratic if m is close to n. On the other hand if $m = n$ you can do it in $3n$ moves (how?), which is linear. Is it always possible to find a linear algorithm? If not, prove it. If yes, write the corresponding algorithm, which works with a global array A of size $n+m+1$, does not use another array and in liner time moves every $a[i]$ to $a[i+m]$ if $i \leq n$ and to $a[i-n]$ if $n < i < m+n+1$.

7. 15 women and 15 men are flying from Earth to Andromeda and should marry themselves. Every man is ready to merry any woman, but women are more restrictive. Every of them have created a list of men, whom she can consider as a possible husband. Those lists are presented in the following table, which describes the set of possible pars. (j in the row Wi means that the marriage between the woman i and the man j is possible).

	can marry with
$W1$	$3, 11, 13$
$W2$	$1, 3, 12$
$W3$	$1, 4, 15$
$W4$	$2, 5, 7, 10$
$W5$	$3, 9, 15$
$W6$	$8, 13, 14$
$W7$	$4, 9, 11, 12$
$W8$	$a, 13, 15$
$W9$	$2, 5, 7, 14$
$W10$	$4, 6, 9, 10$
$W11$	$3, 8, 12$
$W12$	$1, 6, 11, 12$
$W13$	$1, 4, a+b-1$
$W14$	$2, 5, 7, 15$
$W15$	$3, 4, 15$

Is it possible to marry them? (Every man should have one and only one wife). Suggest a general algorithm to solve this problem.

8. Suppose you have a glass ball and want to find what is the highest floor of the 100-floor building from which you can drop the boll and it will be not broken. You need in the worst case 100 experiments. Suppose now that you have two same balls. How many attempts (in the worst case but in the best algorithm) you need to answer the same question? Hint: it is not 50. What about three balls?

9. Given two increasing sequences $x_1 < x_2 < \cdots < x_n$ and $y_1 < y_2 < \cdots < y_n$ show that the sum

$$\sum_{i=1}^{n} x_i y_{\sigma(i)},$$

for σ a permutation of $\{1, 2, \ldots, n\}$, is maximized when σ is the identity permutation ($\sigma(i) = i$ for any i). When the sum is minimized?